Summary: If $\mathcal{C}$ is a class of groups, a $\mathcal{C}$-injector of a finite group $G$ is a subgroup $V$ of $G$ with the property that $V \cap K$ is a $\mathcal{C}$-maximal subgroup of $K$ for all subnormal subgroups $K$ of $G$. The classical result of B. Fischer, W. Gaschütz and B. Hartley states the existence and conjugacy of $\mathfrak{F}$-injectors in finite soluble groups for Fitting classes $\mathfrak{F}$. We shall show that for groups of nilpotent length at most 4, $\mathfrak{F}$-injectors permute with the members of a Sylow basis in the group. We shall exhibit the construction of a Fitting class and a group of nilpotent length 5, which fail to satisfy the result and show that the bound is the best possible.

MSC:

20D10 Finite solvable groups, theory of formations, Schunck classes, Fitting classes, $\pi$-length, ranks
20D20 Sylow subgroups, Sylow properties, $\pi$-groups, $\pi$-structure

Keywords:
finite soluble group; Fitting class; injector; system permutability