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On as-configurations, skew-translation generalised quadrangles of even order and a conjecture of Payne. (English) [Zbl 07413140]
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Summary: Generalised quadrangles, as a special case of generalised polygons, were introduced by Jacques Tits [14] in 1959. About twenty years later Kantor [7] showed that an elation generalised quadrangle is completely characterized by its so-called 4-gonal family or Kantor family.

Motivated by the Ahrens and Szekeres Quadrangle [1], Ghinelli [4] established a variation of the procedure of Kantor [7], by introducing the notion of an AS-configuration of order \( n \). Such a configuration gives rise to a skew-translation generalised quadrangle of order \((n, n)\), and conversely, outlined in J. Bamberg, S.P. Glasby, E. Swartz [2]. The only known groups of even order admitting an AS-configuration are elementary abelian 2-groups. Moreover, Payne [8] conjectured that there are no other examples. Indeed, he found a proof for \( n = 4 \). Moreover it is also true for \( n = 8 \), compare J. Bamberg, S.P. Glasby, E. Swartz [2].

The purpose of this paper is to prove

**Theorem.** A group of even order admitting an AS-configuration of order \( n \) is elementary abelian, if \( n \) is not a square.

In other words, the only skew-translation generalised quadrangles of order \((n, n)\), \( n \equiv 0 \pmod{2} \), are translation generalised quadrangles, if \( n \) is not a square. K. Thas claims in his preprint [12] that he found among other things a proof of Payne’s conjecture. Unfortunately, in the main part of his proof there is a gap not easy to fill, [13].

**MSC:**

51E12 Generalized quadrangles and generalized polygons in finite geometry

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generalised quadrangle