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An optimized multistage complete in phase P-stable algorithm. (English) Zbl 07414710

Summary: A fourteen algebraic order P-stable symmetric four-stages two-step scheme with expunged phase-lag and its first and second derivatives, is developed, for the first time in the literature, in this paper. The new four-stages method is developed based on the following steps:

- Contentment of the necessary and sufficient conditions for P-stability.
- Contentment of the condition of the expunging of the phase-lag.
- Contentment of the junctures of the expunging of the first and second derivatives of the phase-lag.

The result of the above methodology is the development, for the first time in the literature, of a four-stages P-stable fourteen algebraic order symmetric two-step method with expunged phase-lag and its derivatives up to order two.

We present also a full numerical and theoretical analysis for the new algorithm which contains the following steps:

- the development of the new four-stages method,
- the achievement of its local truncation error (LTE),
- the foundation of the asymptotic form of the LTE of the new four-stages method,
- the foundation of the stability and interval of periodicity of the new four-stages method,
- the achievement of an embedded algorithm and the determination of the variable step technique for the changing of the step sizes,
- the evaluation of the computational efficiency of the new four-stages method with its application on:
  - the resonance problem of the radial Schrödinger equation and on
  - the system of the coupled differential equations of the Schrödinger type.

The above study leads to the conclusion that the new four-stages method is more efficient than the existed ones.

MSC:

- 81Q05 Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- 81-08 Computational methods for problems pertaining to quantum theory
- 65L05 Numerical methods for initial value problems involving ordinary differential equations
- 65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations