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Adjacency eigenvalues of graphs without short odd cycles. (English) Zbl 07414958
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Summary: It is well known that spectral Turán type problem is one of the most classical problems in graph theory. In this paper, we consider the spectral Turán type problem. Let $G$ be a graph and let $\mathcal{G}$ be a set of graphs, we say $G$ is $\mathcal{G}$-free if $G$ does not contain any element of $\mathcal{G}$ as a subgraph. Denote by $\lambda_1$ and $\lambda_2$ the largest and the second largest eigenvalues of the adjacency matrix $A(G)$ of $G$, respectively. In this paper we focus on the characterization of graphs without short odd cycles according to the adjacency eigenvalues of the graphs. Firstly, an upper bound on $\lambda_2^{k_1} + \lambda_2^{k_2}$ of $n$-vertex $\{C_3, C_5, \ldots, C_{2k+1}\}$-free graphs is established, where $k$ is a positive integer. All the corresponding extremal graphs are identified. Secondly, a sufficient condition for non-bipartite graphs containing an odd cycle of length at most $2k + 1$ in terms of its spectral radius is given. At last, we characterize the unique graph having the maximum spectral radius among the set of $n$-vertex non-bipartite graphs with odd girth at least $2k + 3$, which solves an open problem proposed by Lin, Ning and Wu [Eigenvalues and triangles in graphs, Combin. Probab. Comput. 30 (2) (2021) 258-270].

MSC:
05Cxx Graph theory
15Axx Basic linear algebra
05-XX Combinatorics

Keywords:
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References:

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