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Platonic solids, Archimedean solids and semi-equivelar maps on the sphere. (English)

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Summary: A map $X$ on a surface is called vertex-transitive if the automorphism group of $X$ acts transitively on the set of vertices of $X$. A map is called semi-equivelar if the cyclic arrangement of faces around each vertex is same. In general, semi-equivelar maps on a surface form a bigger class than vertex-transitive maps. There are semi-equivelar maps on the torus, the Klein bottle and other surfaces which are not vertex-transitive. It is known that the boundaries of Platonic solids, Archimedean solids, regular prisms and anti-prisms are vertex-transitive maps on $S^2$. Here we show that there is exactly one semi-equivelar map on $S^2$ which is not vertex-transitive. As a consequence, we show that all the semi-equivelar maps on $R^2$ are vertex-transitive. Moreover, every semi-equivelar map on $S^2$ can be geometrized, i.e., every semi-equivelar map on $S^2$ is isomorphic to a semi-regular tiling of $S^2$. In the course of the proof of our main result, we present a combinatorial characterisation in terms of an inequality of all the types of semi-equivelar maps on $S^2$. Here we present combinatorial proofs of all the results.

MSC:

52A55 Spherical and hyperbolic convexity
52C22 Tilings in $n$ dimensions (aspects of discrete geometry)
52A40 Inequalities and extremum problems involving convexity in convex geometry

Keywords:

polyhedral maps on sphere; vertex-transitive maps; semi-equivelar maps; semi-regular tilings; Archimedean solids

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References:


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