Summary: Knowledge is a network of interconnected concepts. Yet, precisely how the topological structure of knowledge constrains its acquisition remains unknown, hampering the development of learning enhancement strategies. Here, we study the topological structure of semantic networks reflecting mathematical concepts and their relations in college-level linear algebra texts. We hypothesize that these networks will exhibit structural order, reflecting the logical sequence of topics that ensures accessibility. We find that the networks exhibit strong core-periphery architecture, where a dense core of concepts presented early is complemented with a sparse periphery presented evenly throughout the exposition; the latter is composed of many small modules each reflecting more narrow domains. Using tools from applied topology, we find that the expositional evolution of the semantic networks produces and subsequently fills knowledge gaps, and that the density of these gaps tracks negatively with community ratings of each textbook. Broadly, our study lays the groundwork for future efforts developing optimal design principles for textbook exposition and teaching in a classroom setting.

MSC:
91F20 Linguistics
62R40 Topological data analysis
05C82 Small world graphs, complex networks (graph-theoretic aspects)

Keywords:
persistent homology; mathematics education; learning; network science; semantic networks; applied mathematics; pattern recognition; complexity

References:


