Bridges, Douglas; **Demuth, Osvald**

On the Lebesgue measurability of continuous functions in constructive analysis. (English)

Zbl 0742.03024
Bull. Am. Math. Soc., New Ser. 24, No. 2, 259-276 (1991).

The paper starts with a general discussion of three main approaches to constructive mathematics: that of Bishop (BISH), that of the Russian school (RUSS) and the historically first, i.e. that of Brouwer (INT). The intended reader is a mathematician with little or no knowledge of the technicalities of constructive mathematics. It is argued that RUSS as well as INT can be viewed as BISH with certain adjoined principles. This view makes it possible to prove independence results relative to BISH. The main results of the paper give examples of propositions independent of BISH, e.g.: There is a bounded, pointwise continuous map of [0,1] into $\mathbb{R}$ that is not Lebesgue measurable. The proof depends on a theorem first proved by Demuth in his unpublished thesis in 1964 and later rediscovered by Bridges.

Apart from proving this interesting result the paper gives a good introduction into constructive measure theory, a central part of modern constructive mathematics. It provides detailed references to the literature.

Reviewer: H.Schwichtenberg (München)

MSC:

03F60 Constructive and recursive analysis
03-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to mathematical logic and foundations
03F55 Intuitionistic mathematics
03F65 Other constructive mathematics
28A20 Measurable and nonmeasurable functions, sequences of measurable functions, modes of convergence
03F25 Relative consistency and interpretations

Keywords:

intuitionism; α-singular covering; constructive mathematics; Bishop; Russian school; Brouwer; constructive measure theory

Full Text: DOI

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