Janeczko, S.; **Stewart, Ian**

Symplectic singularities and optical diffraction. (English) Zbl 0742.58020

Singularity theory and its applications. Pt. II: Singularities, bifurcations and dynamics, Proc. Symp., Warwick/UK 1988-89, Lect. Notes Math. 1463, 220-255 (1991).

[For the entire collection see Zbl 0723.00029.]

A manifold P is symplectic if its dimension is even and a closed 2-form ω called the symplectic form of P is defined on it. If (P_1, ω_1) and (P_2, ω_2) are two symplectic manifolds of equal dimension, a smooth map $\psi : P_1 \to P_2$ is said to be symplectic if $\psi^* \omega_2 = \omega_1$, where ψ^* is the pullback of ψ. It can be proved that such a ψ is a local diffeomorphism and since it is symplectic also it is called a local symplectomorphism. When ψ is symplectic and a global diffeomorphism, it is called a global symplectomorphism and then P_1 and P_2 are said to be symplectomorphic. These basic ideas are exploited to develop a symplectic geometry which is a modern description of Hamilton's formalism for a dynamical system.

This well-written work surveys the structure of singularities of symplectic mappings. These singularities play a crucial role in many applications of this theory to the theory of dynamical systems. The first two sections are devoted to developing the theory of symplectic manifolds from first principles after motivating the study by demonstrating the symplectic structure in the trajectory of a particle or a light ray reflected by a smooth curve. In the third section a universal phase space for symplectic geometry is introduced together with the ideas of a symplectic image and a symplectic relation. These ideas are then used to classify stable Lagrangian projections. In section 4 the ideas developed in the preceding sections are used to construct a model for optical instruments. Sections 5 and 6 describe the main results which illustrate the application of symplectic singularities to optics. In particular it is shown that in the study of classical diffraction, a highly singular geometry in the rays results from the projection into physical space of a well behaved structure in phase space. This method is used to analyze diffraction at an aperture and to classify generic caustics by diffraction in a half-line aperture. Finally in section 6 diffraction at a smooth obstacle is considered and canonical varieties of generic obstacle curves in the plane are classified.

Two appendices describe respectively the formulation of classical mechanics in the universal phase space and a detailed description of diffraction at a circular obstacle where the results are particularly interesting.

Reviewer: C.S.Sharma (London)

MSC:

37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
70H03	Lagrange's equations
58-02	Research exposition (monographs, survey articles) pertaining to global analysis

Keywords:
symplectic manifold; singularities; diffraction