Let $S_1 \sim W_p(\Sigma_1, n_1)$ and $S_2 \sim W_p(\Sigma_2, n_2)$ be two independent $p \times p$ Wishart matrices. It is desired to consider the minimax estimation of $(\Sigma_1, \Sigma_2)$ under the loss function

$$\sum_{i=1}^{2} \{ \text{tr}(\Sigma_i^{-1}\hat{\Sigma}_i) - \log |\Sigma_i^{-1}\hat{\Sigma}_i| - p \},$$

extending known results for a single matrix. The aim is to get substantial savings in risk when the eigenvalues of $\Sigma_2\Sigma_1^{-1}$ are close together; this would be useful when there is prior information that the $\Sigma_i$’s are approximately proportional and their eigenvalues are likely to be far apart.

The approach is to first utilize the principle of invariance to narrow the class of estimators under consideration to the equivariant ones. The unbiased estimates of risk of these estimators are then computed and promising estimators are derived from them.

A Monte Carlo study is conducted to evaluate the risk performance of the following estimators: best usual, minimax, adjusted usual, D. K. Dey and C. Srinivasan [see ibid. 13, 1581-1591 (1985; Zbl 0582.62042)], L. R. Haff [ibid. 8, 586-597 (1980; Zbl 0441.62045)], and Stein, where the last three have been proposed by the indicated authors.

Reviewer: R. Mentz (S.M.de Tucuman)

MSC:
62H12 Estimation in multivariate analysis
62F10 Point estimation

Keywords:
covariance matrix; Stein’s loss; equivariance; Wishart matrices; minimax estimation; eigenvalues; principle of invariance; unbiased estimates of risk; Monte Carlo study; risk performance

Full Text: DOI