Summary: We study the singularities of algebraic difference equations on curves from the point of view of equivariant sheaves. We propose a definition for the formal local type of an equivariant sheaf at a point in the case of a reduced curve acted on by a group which is virtually the integers. We show that with this definition, equivariant sheaves can be glued from an “open cover”. Precisely, we show that an equivariant sheaf can be uniquely recovered from the following data: the restriction to the complement of a point, the local type at the point itself, and an isomorphism between the two over the punctured neighborhood of said point. We study symmetric elliptic difference equations (“elliptic equations” from now on) from this point of view. We consider several natural notions for an algebraic version of symmetric elliptic difference equations, i.e. symmetric elliptic difference modules (“elliptic modules”). We show that different versions are not equivalent, but we detail how they are related: all the versions embed fully faithfully into the same category of equivariant sheaves. This implies that we can use the theory for equivariant sheaves to study singularities of elliptic equations as well. One reason to study elliptic equations is that they generalize, and degenerate to, $(q)$-difference equations (i.e. equivariant sheaves) and differential equations (i.e. $D$-modules) on the projective line. We discuss this from the elliptic module point of view, which requires studying elliptic modules on singular curves. We study the relation between elliptic modules on singular curves and their normalization. We show that for modules which are flat at the singular points there is an equivalence and we give examples showing that this cannot be improved upon.

MSC:
14F43 Other algebro-geometric (co)homologies (e.g., intersection, equivariant, Lawson, Deligne (co)homologies)
14B20 Formal neighborhoods in algebraic geometry

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References: