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Summary: This study aims to develop a mathematical model that can describe the vertical distribution of suspended sediment particles in an open channel turbulent flow under unsteady nonequilibrium condition with non-local mixing effect. A space non-local transport of sediment is considered that arises due to turbulent bursting where the hopping height of a particle is not limited to a small distance as described by classical theory of turbulence, rather it can jump up to any height. To take into account this fact unlike previous researchers who used Rouse equation, Hunt equation or traditional advection-diffusion equation as the governing equation, the present study uses fractional advection-diffusion equation (fADE) where space fractional derivative for the local diffusive flux has been used that counts the effect of non-locality. The fractional derivative is considered in Caputo sense and the order $\alpha$ of the fractional derivative corresponds to $\alpha$-order Lévy stable distribution with value $1 < \alpha \leq 2$. The non-local effect has been considered in the boundary conditions as well as in the non-linear models of sediment diffusivity. Four types of sediment diffusivity are considered which are generalized non-linear models for broad applicability of the study. The fADE together with the boundary and initial conditions, is solved by Chebyshev collocation method and Euler backward method. This proposed method is unconditionally convergent and converges more rapidly than other previous methods used in similar studies. Effects of non-local mixing have been investigated for transient and bottom concentration distributions. Proposed models have been validated for sediment distribution under steady and unsteady condition with existing experimental data and satisfactory results are obtained for all choices of sediment diffusivity. The variation of transient and bottom concentration with non-local effects are physically justified. Validation results show that the model can be applied to describe vertical concentration distribution in unsteady and steady turbulent flows in practical situation.

MSC: 76Fxx Turbulence

Keywords: fractional advection-diffusion equation (fADE); suspension concentration distribution; unsteady turbulent flow; Chebyshev polynomial of first kind; Chebyshev collocation method

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