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Semigenerated Carnot algebras and applications to sub-Riemannian perimeter. (English)
Math. Z. 299, No. 3-4, 2257-2285 (2021)

Summary: This paper contributes to the study of sets of finite intrinsic perimeter in Carnot groups. Our intent is to characterize in which groups the only sets with constant intrinsic normal are the vertical half-spaces. Our viewpoint is algebraic: such a phenomenon happens if and only if the semigroup generated by each horizontal half-space is a vertical half-space. We call semigenerated those Carnot groups with this property. For Carnot groups of nilpotency step 3 we provide a complete characterization of semigeneration in terms of whether such groups do not have any Engel-type quotients. Engel-type groups, which are introduced here, are the minimal (in terms of quotients) counterexamples. In addition, we give some sufficient criteria for semigeneration of Carnot groups of arbitrary step. For doing this, we define a new class of Carnot groups, which we call type (♢) and which generalizes the previous notion of type (⋆) defined by M. Marchi. As an application, we get that in type (♢) groups and in step 3 groups that do not have any Engel-type algebra as a quotient, one achieves a strong rectifiability result for sets of finite perimeter in the sense of Franchi, Serapioni, and Serra-Cassano.

MSC:

22E15 General properties and structure of real Lie groups
53C17 Sub-Riemannian geometry
22A15 Structure of topological semigroups
22E25 Nilpotent and solvable Lie groups
28A75 Length, area, volume, other geometric measure theory
49Q15 Geometric measure and integration theory, integral and normal currents in optimization
22A15 Structure of topological semigroups
22E15 General properties and structure of real Lie groups

Keywords:
Carnot algebra; horizontal half-space; semigroup generated; Lie wedge; constant intrinsic normal; finite sub-Riemannian perimeter; Engel-type algebras; type diamond; trimmed algebra

Full Text: DOI

References:


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