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Sumset phenomenon in locally compact topological groups. (English) [Zbl 07429993]
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In this article the authors prove Jin’s Theorem for amenable non-compact locally compact topological groups using the ultrafilter method of M. Beiglböck.

Let be $G$ a locally compact non-compact topological group and let $CB(G)$ denote the collection of all bounded complex valued continuous functions on $G$ with the uniform norm. A function $f \in CB(G)$ is a left norm continuous if $g \mapsto f \circ \lambda_g : G \to CB(G)$ is norm continuous, where $\lambda_g$ denotes the left translation by $g \in G$. The collection of all left norm continuous functions on $G$ is an $m$-admissible $C^*$-subalgebra of $CB(G)$ and is denoted by $Luc(G)$. For $f \in Luc(G)$, $f^{-1}(\{0\})$ is called a zero set.

The collection of all compact subsets of $G$ with positive Haar measure $m$ is denoted by $P_\mu(G)$. Let $F = \{F_n\}_{n \in \mathbb{N}}$ be a net in $P_\mu(G)$. The upper Banach density of $A \subseteq G$ is defined by $d^*_F(A) = \sup\{\alpha : (\forall k \in D)(\exists n \geq k)(\exists g \in G)(m^*(A \cap F_n g) \geq \alpha m(F_n))\}$, where $m^*$ is the outer measure of $m$. The net $F$ is a (left) Følner net if and only if for each $g \in G$, the net $\left\{\frac{m(gF_n A F_n)}{m(F_n)}\right\}_{n \in \mathbb{N}}$ converges to 0. The group $G$ is called amenable if there exists a sequence of compact subsets of $G$ that is Følner.

The following result plays an important role in this work, which is a weak version of Furstenberg’s Principal Theorem.

- If $A$ is a closed subset of $G$ such that $d^*_F(A) > 0$, then $F$ is a Følner net in $P_\mu(G)$, then there is a countably additive regular measure $\mu$ on the set $B$ of Borel subsets of $G^{Luc}$ such that
  \[ 1 \) $\mu(\overline{A}) = d^*_F(A)$ ($\overline{A}$ denotes the closure of $A$ in $G^{Luc}$),
  \[ 2 \) for all closed subsets $B$ of $G$, $\mu(\overline{B}) \leq d^*_F(B)$,
  \[ 3 \) for all $B \in \mathcal{B}$ and all $g \in G$, $\mu(gB) = \mu(B)$, and
  \[ 4 \) $\mu(G^{Luc}) = 1$.

Some Ramsey Theoretic results have been obtained too. Finally the authors state Jin’s Theorem.

- Let $G$ be a $\sigma$-compact non-compact amenable topological group, then there exists left and right Følner sequences in $P_\mu(G)$, and so it can define notions $d^L_F$ and $d^R_F$ as left and right Banach density, respectively.

Let $A$ and $B$ be two zero subsets of $G$ such that $d^L_F(A) d^R_F(B) > 0$, then $B^{-1}A$ is (right) piecewise syndetic. (A subset $A$ of $G$ is called thick if and only if for every finite subset $F$ of $G$ there exists $g \in G$ such that $Fg \subseteq A$, and $A$ is piecewise syndetic if there exists a finite subset $H$ of $G$ such that $\bigcup_{h \in H} h^{-1}A$ is thick.)

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MSC:

- 22B05 General properties and structure of LCA groups
- 37A15 General groups of measure-preserving transformations and dynamical systems
- 11B05 Density, gaps, topology

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References:


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