Let $G$ be a graph and $c_R : E(G) \to [k]$ be an edge-coloring of $G$. A rainbow path between $u, v \in V(G)$ is a path $P$ from $u$ to $v$ such that for all $e, e' \in E(P)$, where $e \neq e'$, we have $c_R(e) \neq c_R(e')$. In the Rainbow $k$-Coloring problem for a given graph $G$ the objective is to decide if there exists an edge-coloring $c_R : E(G) \to [k]$ such that for all $u, v \in V(G)$ there is a rainbow path between $u$ and $v$ in $G$. Two variants of the above problem are Subset Rainbow $k$-Coloring and Steiner Rainbow $k$-Coloring, where for a given subset $S \subseteq V(G) \times V(G)$ respectively $S' \subseteq V(G)$ the objective is to check if there is $c_R : E(G) \to [k]$ such that there is a rainbow path for each $(u, v) \in S$ respectively $u, v \in S'$.

In this paper it is proved that for all $k \geq 3$, Rainbow $k$-Coloring does not admit an algorithm running in time $2^{o(|E(G)|)}n^{O(1)}$ unless ETH (Exponential Time Hypothesis) fails. This (partially) solves a conjecture of Ł. Kowalik et al. [LIPIcs – Leibniz Int. Proc. Inform. 57, Article 58, 16 p. (2016; Zbl 1397.68102)]. The author also studies the problem Steiner Rainbow $k$-Coloring and proves that for every $k \geq 3$ the problem does not admit an algorithm running in time $2^{o((|S|^2)n^{O(1)})}$ unless ETH fails. She also obtains that for each $k \geq 3$, Subset Rainbow $k$-Coloring and Steiner Rainbow $k$-Coloring admit $2^{O(|S|)n^{O(1)}}$- and $2^{O((|S|^2)n^{O(1)})}$-time algorithms, respectively. Three open directions of study conclude the paper.

Reviewer: Ioan Tomescu (Bucureşti)

MSC:
68R10 Graph theory (including graph drawing) in computer science
05C15 Coloring of graphs and hypergraphs
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)
68Q25 Analysis of algorithms and problem complexity

Keywords:
rainbow coloring; lower bound; ETH; fine-grained complexity

Full Text: DOI

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