Brasco, Lorenzo; De Philippis, Guido; Franzina, Giovanni  
*Positive solutions to the sublinear Lane-Emden equation are isolated.*  

The authors establish the following fine and important property of positive solutions of the sublinear Lane-Emden problem $-\Delta u = |u|^{q-2}u$ in a bounded open set $\Omega$ with zero Dirichlet boundary conditions on $\partial\Omega$, $1 < q < 2$. Namely, it is proved that, under certain regularity assumptions on $\Omega$, the unique positive minimizer $w$ of the energy functional $\frac{1}{2} \int_{\Omega} |\nabla u|^2 \, dx - \frac{1}{q} \int_{\Omega} |u|^q \, dx$ in $H^1_0(\Omega)$ is isolated in the $L^1(\Omega)$-norm topology. As a consequence, the least generalized frequency $\lambda_1 = \min\{\int_{\Omega} |\nabla u|^2 \, dx : \int_{\Omega} |u|^q \, dx = 1\}$ is isolated in the sense that the second smallest frequency $\lambda_2$ satisfies $\lambda_1 < \lambda_2$. In contrast, if required regularity assumptions on $\Omega$ are violated, $\lambda_1$ might not be isolated, see [L. Brasco and G. Franzina, Adv. Nonlinear Anal. 8, 707–714 (2019; Zbl 1412.35209)]. As the main auxiliary tool for the proof, the authors comprehensively study the weighted embedding $H^1_0(\Omega) \hookrightarrow L^q(\Omega; w^{-2})$.

Reviewer: Vladimir Bobkov (Ufa)

**MSC:**

- [35P30](https://zbmath.org/classification/35P30) Nonlinear eigenvalue problems and nonlinear spectral theory for PDEs  
- [35B38](https://zbmath.org/classification/35B38) Critical points of functionals in context of PDEs (e.g., energy functionals)  
- [35J61](https://zbmath.org/classification/35J61) Semilinear elliptic equations  
- [35R09](https://zbmath.org/classification/35R09) Integro-partial differential equations  
- [49R05](https://zbmath.org/classification/49R05) Variational methods for eigenvalues of operators  
- [58E05](https://zbmath.org/classification/58E05) Abstract critical point theory (Morse theory, Lyusternik-Shnirel’man theory, etc.) in infinite-dimensional spaces

**Keywords:**

- cone condition; constrained critical points; Lane-Emden equation; first eigenvalue

**Full Text:** DOI

**References:**


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