Pellegrino, Daniel; Raposo, Anselmo

Constants of the Kahane-Salem-Zygmund inequality asymptotically bounded by 1. (English)


Summary: The Kahane-Salem-Zygmund inequality for multilinear forms in $\ell_\infty$ spaces claims that, for all positive integers $m, n_1, \ldots, n_m$, there exists an $m$-linear form $A : \ell_\infty^{n_1} \times \cdots \times \ell_\infty^{n_m} \rightarrow \mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) of the type

$$A(z^{(1)}, \ldots, z^{(m)}) = \sum_{j_1 = 1}^{n_1} \cdots \sum_{j_m = 1}^{n_m} \pm z^{(1)}_{j_1} \cdots z^{(m)}_{j_m},$$

satisfying

$$\|A\| \leq C_m \max\{n_1^{1/2}, \ldots, n_m^{1/2}\} \prod_{j=1}^{m} n_j^{1/2},$$

for

$$C_m \leq \kappa \sqrt{m \log m \sqrt{m!}}$$

and a certain $\kappa > 0$. Our main result shows that given any $\epsilon > 0$ and any positive integer $m$, there exists a positive integer $N$ such that

$$C_m < 1 + \epsilon,$$

when we consider $n_1, \ldots, n_m > N$. In addition, while the original proof of the Kahane-Salem-Zygmund relies on highly non-deterministic arguments, our approach is constructive. We also provide the same asymptotic bound (which is shown to be optimal in some cases) for the constant of a related non-deterministic inequality proved by G. Bennett in 1977. Applications to Berlekamp’s switching game are given.

MSC:

47A07 Forms (bilinear, sesquilinear, multilinear)
15A60 Norms of matrices, numerical range, applications of functional analysis to matrix theory
15A51 Stochastic matrices (MSC2000)

Keywords:

multilinear forms; Hadamard matrices; sequence spaces; Berlekamp’s switching game

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.