Summary: The system $S_c(L)$ consisting of joins of closed sublocales of a locale $L$ is known to be a frame, and for $L$ subfit it coincides with the Booleanization $S_b(L)$ of the coframe of sublocales of $L$. In this paper, we study $S_b(L)$ for a general locale $L$. We show that $S_c(L)$ is always a subframe of $S_b(L)$. Moreover, if $X$ is a $T_D$-space, we prove that $S_c(\Omega(X))$ is precisely the set of classical subspaces of $X$, and that a locale $L$ is $T_D$-spatial iff the Boolean algebra $S_b(L)$ is atomic. Some functoriality properties of $S_b(L)$ are also studied.

MSC:

- 18F70 Frames and locales, pointfree topology, Stone duality
- 06D22 Frames, locales
- 54D35 Extensions of spaces (compactifications, supercompactifications, completions, etc.)
- 54D10 Lower separation axioms ($T_0$–$T_3$, etc.)

Keywords:

- locale; frame; sublocale; Booleanization; induced sublocale; complemented sublocale; subfit locale; $T_D$-axiom

Full Text: DOI

References:


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