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Iterative splitting schemes for a soft material poromechanics model. (English) Zbl 07442763
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Summary: We address numerical solvers for a poromechanics model particularly adapted for soft materials, as it generally respects thermodynamics principles and energy balance. Considering the multi-physics nature of the problem, which involves solid and fluid species, interacting on the basis of mass balance and momentum conservation, we decide to adopt a solution strategy of the discrete problem based on iterative splitting schemes. As the model is similar (but not equivalent to) the Biot poromechanics problem, we follow the abundant literature for solvers of the latter equations, developing two approaches that resemble the well known undrained and fixed-stress splits for the Biot model. A thorough convergence analysis of the proposed schemes is performed. In particular, the undrained-like split is developed and analyzed in the framework of generalized gradient flows, whereas the fixed-stress-like split is understood as block-diagonal L^2 -type stabilization and analyzed by means of a relative stability analysis. In addition, the application of Anderson acceleration is suggested, improving the robustness of the split schemes. Finally, we test these methods on different benchmark tests, and we also compare their performance with respect to a monolithic approach. Together with the theoretical analysis, the numerical examples provide guidelines to appropriately choose what split scheme shall be used to address realistic applications of the soft material poromechanics model.

MSC:

74-XX Mechanics of deformable solids

76-XX Fluid mechanics

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Keywords:

poromechanics of soft materials; iterative splitting schemes; undrained split; fixed-stress split; convergence analysis

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