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The Wiener index of signed graphs. (English) Zbl 07442836

Summary: The Wiener index of a graph $W(G)$ is a well studied topological index for graphs. An outstanding problem of Šoltés is to find graphs $G$ such that $W(G) = W(G - v)$ for all vertices $v \in V(G)$, with the only known example being $G = C_{11}$. We relax this problem by defining a notion of Wiener indices for signed graphs, which we denote by $W_{\sigma}(G)$, and under this relaxation we construct many signed graphs such that $W_{\sigma}(G) = W_{\sigma}(G - v)$ for all $v \in V(G)$. This ends up being related to a problem of independent interest, which asks when it is possible to 2-color the edges of a graph $G$ such that there is a path between any two vertices of $G$ which uses each color the same number of times. We briefly explore this latter problem, as well as its natural extension to $r$-colorings.

MSC:
05Cxx Graph theory
92Exx Chemistry
51-XX Geometry

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