The question of existence and uniqueness of solutions to the Yang-Mills heat equation is of intrinsic interest, partly because it is a naturally occurring quasilinear diffusion equation and partly because of the way that gauge invariance intervenes in the very formulation of the Cauchy problem. But, as in [N. Charalambous and L. Gross, Commun. Math. Phys. 317, No. 3, 727–785 (2013; Zbl 1279.58005); J. Math. Phys. 56, No. 7, 073505, 21 p. (2015; Zbl 1321.81041)], this work is ultimately aimed at the construction of gauge invariant functions of distributional initial data by a completion procedure sketched in the introduction to [N. Charalambous and L. Gross, Commun. Math. Phys. 317, No. 3, 727–785 (2013; Zbl 1279.58005)], in an anticipated application to quantum field theory. In order to construct local observables for this application the author is interested in solutions over a bounded open subset of $\mathbb{R}^3$, as well as over all of $\mathbb{R}^3$. The question of boundary conditions therefore arises. In [loc. cit.], Dirichlet, Neumann and Marini boundary conditions are considered. The latter consists in setting the normal component of the curvature to zero on the boundary. These are the only boundary conditions commensurate with the intended applications to quantum field theory. As the author mentioned, solutions satisfying Marini boundary conditions are derived from solutions satisfying Neumann boundary conditions in a future work. In this paper the author only consider Dirichlet and Neumann boundary conditions. He proves the existence and uniqueness of solutions to the Yang-Mills heat equation over $\mathbb{R}^3$ and over a bounded open convex set in $\mathbb{R}^3$. The initial data is taken to lie in the Sobolev space of order one half, which is the critical Sobolev index for this equation over a three dimensional manifold. The existence is proven by solving first an augmented, strictly parabolic equation and then gauge transforming the solution to a solution of the Yang-Mills heat equation itself. The gauge functions needed to carry out this procedure lie in the critical gauge group of Sobolev regularity three halves, which is a complete topological group in a natural metric but is not a Hilbert Lie group. The nature of this group must be understood in order to carry out the reconstruction procedure. Solutions to the Yang-Mills heat equation are shown to be strong solutions modulo these gauge functions. Energy inequalities and Neumann domination inequalities are used to establish needed initial behavior properties of solutions to the augmented equation. This paper is organized as follows: Chapter 1 is an introduction to the subject. In Chapter 2, the author states the results obtained in this paper. Chapter 3 deals with solutions for the augmented Yang-Mills heat equation. Chapter 4 is devoted to initial behavior of solutions to the augmented equation, and Chapter 5 to gauge groups. Chapter 6 deals with the conversion group. Chapter 7 is devoted to the prove of the main result.

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MSC:
35-02 Research exposition (monographs, survey articles) pertaining to partial differential equations
35K58 Semilinear parabolic equations
35K65 Degenerate parabolic equations
70S15 Yang-Mills and other gauge theories in mechanics of particles and systems
35K51 Initial-boundary value problems for second-order parabolic systems
58J35 Heat and other parabolic equation methods for PDEs on manifolds

Keywords:
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References:


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