Summary: Let $H$ be a separable infinite-dimensional complex Hilbert space, $\mathcal{B}(H)$ the algebra of bounded linear operators acting on $H$ and $\mathcal{J}$ a proper two-sided ideal of $\mathcal{B}(H)$. Denote by $U_\mathcal{J}(H)$ the group of all unitary operators of the form $I + \mathcal{J}$. Recall that an operator $A \in \mathcal{B}(H)$ is diagonalizable if there exists a unitary operator $U$ such that $UAU^*$ is diagonal with respect to some orthonormal basis. A more restrictive notion of diagonalization can be formulated with respect to a fixed orthonormal basis $e = \{e_n\}_{n \geq 1}$ and a proper operator ideal $\mathcal{J}$ as follows: $A \in \mathcal{B}(H)$ is called restrictedly diagonalizable if there exists $U \in U_\mathcal{J}(H)$ such that $UAU^*$ is diagonal with respect to $e$. In this work we give a sufficient condition for a diagonalizable operator to be restrictedly diagonalizable. This condition becomes a characterization when the ideal is arithmetic mean closed. Then we obtain results on the structure of the set of all restrictedly diagonalizable operators. In this way we answer several open problems recently raised by Beltiţă, Patnaik and Weiss.

MSC:

22E65 Infinite-dimensional Lie groups and their Lie algebras: general properties
47B10 Linear operators belonging to operator ideals (nuclear, $p$-summing, in the Schatten-von Neumann classes, etc.)
47A53 (Semi-) Fredholm operators; index theories

Keywords:
operator ideal; restricted diagonalization; essential codimension

Full Text: DOI

References:


[34] Riesz, F.; Sz.-Nagy, B., Leçons d’analyse fonctionnelle (1952), Budapest


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.