Let $V$ be a vector space of dimension $N$ over the finite field $\mathbb{F}_q$ and $T$ be a linear operator on $V$. Given an integer $m$ that divides $N$, an $m$-dimensional subspace $W$ of $V$ is $T$-splitting if $V = W \oplus TW \oplus \cdots \oplus T^{d-1}W$ where $d = N/m$. Let $\sigma(m, d; T)$ denote the number of $m$-dimensional $T$-splitting subspaces. Determining $\sigma(m, d; T)$ for an arbitrary operator $T$ is an open problem. We prove that $\sigma(m, d; T)$ depends only on the similarity class type of $T$ and give an explicit formula in the special case where $T$ is cyclic and nilpotent. Denote by $\sigma_q(m, d; \tau)$ the number of $m$-dimensional splitting subspaces for a linear operator of similarity class type $\tau$ over an $\mathbb{F}_q$-vector space of dimension $md$. For fixed values of $m, d$ and $\tau$, we show that $\sigma_q(m, d; \tau)$ is a polynomial in $q$.

MSC:
11T06 Polynomials over finite fields
05A15 Exact enumeration problems, generating functions
11T99 Finite fields and commutative rings (number-theoretic aspects)
15B33 Matrices over special rings (quaternions, finite fields, etc.)
05A05 Permutations, words, matrices

Keywords:
splitting subspace; Krylov space; anti-invariant subspace; invariant subspace lattice; $q$-Vandermonde identity; finite field

Full Text: DOI

References:


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