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Volume growth, curvature, and buser-type inequalities in graphs. (English) [Zbl 07458135]

Authors’ abstract: We study the volume growth of metric balls as a function of the radius in discrete spaces and focus on the relationship between volume growth and discrete curvature. We improve volume growth bounds under a lower bound on the so-called Ollivier curvature and discuss similar results under other types of discrete Ricci curvature.

Following recent work in the continuous setting of Riemannian manifolds (by the 1st author [Int. Math. Res. Not. 2015, No. 16, 7510–7551 (2015; Zbl 1328.58027)]), we then bound the eigenvalues of the Laplacian of a graph under bounds on the volume growth. In particular, $\lambda_2$ of the graph can be bounded using a weighted discrete Hardy inequality and the higher eigenvalues of the graph can be bounded by the eigenvalues of a tridiagonal matrix times a multiplicative factor, both of which only depend on the volume growth of the graph. As a direct application, we relate the eigenvalues to the Cheeger isoperimetric constant. Using these methods, we describe classes of graphs for which the Cheeger inequality is tight on the 2nd eigenvalue (i.e. the 1st nonzero eigenvalue). We also describe a method for proving Buser’s Inequality [P. Buser, Ann. Sci. Éc. Norm. Supér. (4) 15, 213–230 (1982; Zbl 0501.53030)] in graphs, particularly under a lower bound assumption on curvature.

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MSC:
53A70 Discrete differential geometry
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graphs; discrete Laplacian operators; eigenvalues’ upper bounds; Ollivier curvature; Cheeger isoperimetric constant; Buser’s Inequality

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