Mařík, Jan; **Weil, Clifford E.**

Sums of powers of derivatives. (English) Zbl 0746.26002

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The authors prove analogous results for sums of powers of positive derivatives as they proved for products in the paper in Trans. Am. Math. Soc. 276, 361-373 (1983; Zbl 0517.26006). I mention only the last Theorem:

Let $p \in (1,\infty)$, let $f_1,\dots,f_m \in M$, let $\varphi = \|(f_1,\dots,f_m)\|_p$, where $\|.\|_p$ denotes the p-norm on R^m and $\phi = \varphi^p$. Let us suppose that $\liminf_{y\to t} \text{ap}\ \varphi(y) > 0$ for each $t \in R$. Then the following conditions are equivalent: (i) $\phi \in Q_p$, where Q_p is the system of all functions Ω with the following property: there exist a natural number r, positive numbers q_j and nonnegative derivatives h_j, $j \in \{1,\dots,r\}$, such that $q_1+\dots+q_r \le p$ and $\Omega = h_1^{q_1}\cdots h_r^{q_r}$;

(ii) $\varphi \in D$, where D is the system of all derivatives on R;

(iii) there exist approximately continuous functions $\alpha_1,\dots,\alpha_m$ such that $f_1 = \alpha_1\varphi,\dots,f_m = \alpha_m\varphi$;

(iv) there exist functions ψ, $\alpha_1,\dots,\alpha_m$ such that ψ is a non-negative derivative, $\alpha_1,\dots,\alpha_m$ are approximately continuous and $f_1 = \alpha_1\psi,\dots,f_m = \alpha_m\psi$;

(v) $\varphi \in M$, where M is the system of all derivatives f on R such that fg is a derivative for any bounded approximately continuous function g.

At the end of the paper there are three counterexamples.

Reviewer: L.Mišík (Bratislava)

MSC:

26A24	Differentiation (real functions of one variable): general theory, generalized derivatives, mean value theorems	
26A15	Continuity and related questions (modulus of continuity, semicontinuity, discontinuities, etc.) for real functions in one variable	

Keywords:

nonnegative derivatives; approximate continuity; sums of powers of positive derivatives; approximately continuous functions

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