Hamilton, David H.


The author has as his main theme the dependence of solutions to the Beltrami equation \( \bar{F} = \mu F \) and connections to Teichmüller theory and harmonic analysis. He considers the function \( \mu \) with \( \|\mu\|_\infty < 1 \), sometimes with compact support, and lets \( F^\mu \) be a “normalized” solution of \( (*) \). Theorem 1 shows that the map \( \mu \rightarrow \log(F^\mu) \) is a complex holomorphic map of BMO(\( \mathbb{R}^2 \)). This is an extension of a well-known result of H. M. Reimann [Comment. Math. Helv. 49, 260-276 (1974; Zbl 0289.30027)] but uses careful analysis of the argument of \( (F^\mu)_z \), via approximation.

An open set \( \Omega \ni \infty \) has a univalence criterion if there is an \( a = a(\Omega) \) such that if \( g \) is analytic in \( \Omega \) with \( |zg(z)| = o(1) \) at \( \infty \) and \( |g'(z)| \text{dist}(z, \partial \Omega) < \infty \) then \( g \) is one-to-one. Theorem 2 asserts that this is equivalent to several things, one being that \( g \) has a representation as a Hilbert transform of a function \( h \in L^\infty(\Omega^c) \), and makes contact with the improved Thurston- Sullivan \( \lambda \)-lemma [cf. L. Bers and H. L. Royden, Acta Math. 157, 259-286 (1986; Zbl 0619.30027)]. These results require no regularity of \( \partial \Omega \).

Analogues of some results are given for VMO. The paper is compactly written, and has a few inessential typographical errors.

**MSC:**

- 30C65 Quasiconformal mappings in \( \mathbb{R}^n \), other generalizations
- 30D55 \( H^p \)-classes (MSC2000)
- 30F60 Teichmüller theory for Riemann surfaces

**Keywords:**

Beltrami equation; Teichmüller theory; VMO

**Full Text:** DOI