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BMO and Teichmüller space. (English) Zbl 0746.30021

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The author has as his main theme the dependence of solutions to the Beltrami equation (*) $F_{\bar{z}} = \mu F_z$ and connections to Teichmüller theory and harmonic analysis. He considers the function $\mu(z)$ with $\|\mu\|_\infty < 1$, sometimes with compact support, and lets F^μ be a "normalized" solution of (*). Theorem 1 shows that the map $\mu \to \log(F_z^\mu)$ is a complex holomorphic map of $\mathrm{BMO}(\mathbb{R}^2)$. This is an extension of a well-known result of *H. M. Reimann* [Comment. Math. Helv. 49, 260-276 (1974; Zbl 0289.30027)] but uses careful analysis of the argument of $(F^\mu)_z$, via approximation.

An open set $\Omega \ni \infty$ has a univalence criterion if there is an $a = a(\Omega)$ such that if g is analytic in Ω with $|zg(z)| = o(1)$ at ∞ and $|g'(z)|\mathrm{dist}(z, \partial\Omega) < \infty$ then g is one-to-one. Theorem 2 asserts that this is equivalent to several things, one being that g has a representation as a Hilbert transform of a function $h \in L^\infty(\Omega^c)$, and makes contact with the improved Thurston- Sullivan λ-lemma [cf. *L. Bers* and *H. L. Royden*, Acta Math. 157, 259-286 (1986; Zbl 0619.30027)]. These results require no regularity of $\partial\Omega$.

Analogues of some results are given for VMO. The paper is compactly written, and has a few inessential typographical errors.

MSC:

30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations
30D55 H^p-classes (MSC2000)
30F60 Teichmüller theory for Riemann surfaces

Keywords:

Beltrami equation; Teichmüller theory; VMO

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