Ceccherini-Silberstein, Tullio; Coornaert, Michel; Phung, Xuan Kien
On linear shifts of finite type and their endomorphisms. (English) J. Pure Appl. Algebra 226, No. 6, Article ID 106962, 27 p. (2022)

Summary: Let $G$ be a group and let $A$ be a finite-dimensional vector space over an arbitrary field $K$. We study finiteness properties of linear subshifts $\Sigma \subset A^G$ and the dynamical behavior of linear cellular automata $\tau : \Sigma \to \Sigma$. We say that $G$ is of $K$-linear Markov type if, for every finite-dimensional vector space $A$ over $K$, all linear subshifts $\Sigma \subset A^G$ are of finite type. We show that $G$ is of $K$-linear Markov type if and only if the group algebra $K[G]$ is one-sided Noetherian. We prove that a linear cellular automaton $\tau$ is nilpotent if and only if its limit set, i.e., the intersection of the images of its iterates, reduces to the zero configuration. If $G$ is infinite, finitely generated, and $\Sigma$ is topologically mixing, we show that $\tau$ is nilpotent if and only if its limit set is finite-dimensional. A new characterization of the limit set of $\tau$ in terms of pre-injectivity is also obtained.

MSC:
37B15 Dynamical aspects of cellular automata
37B20 Notions of recurrence and recurrent behavior in topological dynamical systems
37B51 Multidimensional shifts of finite type
20F65 Geometric group theory
68Q80 Cellular automata (computational aspects)

Keywords:
linear subshift of finite type; linear cellular automaton; polycyclic group; group of linear Markov type; Noetherian group algebra; limit set of nilpotent cellular automaton

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References: