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Critical values of homology classes of loops and positive curvature. (English) Zbl 07467759

Summary: We study compact and simply-connected Riemannian manifolds \((M, g)\) with positive sectional curvature \(K \geq 1\). For a nontrivial homology class of lowest positive dimension in the space of loops based at a point \(p \in M\) or in the free loop space one can define a critical length \(\text{crl}_p(M, g)\) resp. \(\text{crl}(M, g)\). Then \(\text{crl}_p(M, g)\) equals the length of a geodesic loop with base point \(p\) and \(\text{crl}(M, g)\) equals the length of a closed geodesic. This is the idea of the proof of the existence of a closed geodesic of positive length presented by Birkhoff in case of a sphere and by Lusternik & Fet in the general case. It is the main result of the paper that the numbers \(\text{crl}_p(M, g)\) resp. \(\text{crl}(M, g)\) attain its maximal value \(2\pi\) only for the round metric on the \(n\)-sphere.

Under the additional assumption \(K \leq 4\) this result for \(\text{crl}(M, g)\) follows from results by Sugimoto in even dimensions and Ballmann, Thorbergsson & Ziller in odd dimensions.

MSC:
53C20 Global Riemannian geometry, including pinching
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
53C22 Geodesics in global differential geometry
53C24 Rigidity results
58E10 Variational problems in applications to the theory of geodesics (problems in one independent variable)
53-XX Differential geometry

Keywords:
free loop space; geodesic loops; loop space; Morse theory; positive sectional curvature

Full Text: DOI Link

References:


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