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For a set $X$ in the $n$-dimensional Euclidean space $\mathbb{R}^n$, an embedding $f: X \rightarrow \mathbb{R}^n$ is called quasisymmetric if there is a homeomorphism $c: [0, \infty) \rightarrow [0, \infty)$ such that $|f(y) - f(x)| \leq c(r)|f(z) - f(x)|$ for all $x, y, z \in X$ with $|y - x| \leq r|z - x|$. In the case $n = 2$, it is well known that every quasisymmetric embedding of a closed disc $B^2$ into $\mathbb{R}^2$ can be extended to a quasiconformal automorphism of $\mathbb{R}^2$.

On the other hand, F. W. Gehring [Tr. Mezhdunarod. Kongr. Mat., Moskva 1966, 313–318 (1968; Zbl 0193.03803)] proved that there are quasisymmetric embeddings of a closed ball $B^3$ into $\mathbb{R}^3$ which cannot be extended to embeddings of an open neighborhood $U$ of $B^3$.

In this paper, the author constructs a quasisymmetric embedding of a closed ball $B$ into $\mathbb{R}^3$ which is quasiconformal inside $B$ and cannot be extended to an embedding of any neighborhood of any boundary point of $B$. In his argument he constructs a geometrically finite Kleinian group acting on $\mathbb{R}^3$ whose limit set is a wildly knotted sphere and uses an ingenious construction of the spherical covering.

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MSC:

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