Summary: For every tuple $d_1, \ldots, d_l \geq 2$, let $\mathbb{R}^{d_1} \otimes \cdots \otimes \mathbb{R}^{d_l}$ denote the tensor product of $\mathbb{R}^{d_i}$, $i = 1, \ldots, l$. Let us denote by $\mathcal{B}(d)$ the hyperspace of centrally symmetric convex bodies in $\mathbb{R}^d$, $d = d_1 \cdots d_l$, endowed with the Hausdorff distance, and by $\mathcal{B}_\otimes(d_1, \ldots, d_l)$ the subset of $\mathcal{B}(d)$ consisting of the convex bodies that are closed unit balls of reasonable crossnorms on $\mathbb{R}^{d_1} \otimes \cdots \otimes \mathbb{R}^{d_l}$. It is known that $\mathcal{B}_\otimes(d_1, \ldots, d_l)$ is a closed, contractible and locally compact subset of $\mathcal{B}(d)$. The hyperspace $\mathcal{B}_\otimes(d_1, \ldots, d_l)$ is called the space of tensorial bodies. In this work we determine the homeomorphism type of $\mathcal{B}_\otimes(d_1, \ldots, d_l)$. We show that even if $\mathcal{B}_\otimes(d_1, \ldots, d_l)$ is not closed with respect to the Minkowski sum, it is an absolute retract homeomorphic to $Q \times \mathbb{R}^p$, where $Q$ is the Hilbert cube and $p = \frac{d_1(d_1+1)+\cdots+d_l(d_l+1)}{2}$. Among other results, the relation between the Banach-Mazur compactum and the Banach-Mazur type compactum associated to $\mathcal{B}_\otimes(d_1, \ldots, d_l)$ is examined.

MSC:
52A07 Convex sets in topological vector spaces (aspects of convex geometry)
46A55 Convex sets in topological linear spaces; Choquet theory
46B28 Spaces of operators; tensor products; approximation properties
57N16 Geometric structures on manifolds of high or arbitrary dimension

Keywords:
infinite-dimensional topology; convex body; tensor norm; hyperspace; tensor product of convex sets; Banach-Mazur compactum

Full Text: DOI arXiv

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