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On the extended Clark-Wormald Hamiltonian-like index problem. (English) [Zbl 07473514]

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Summary: For a Hamiltonian property $\mathcal{P}$, L. H. Clark and N. C. Wormald [Ars Comb. 15, 131–148 (1983; Zbl 0536.05046)] introduced the problem of investigating the value $\mathcal{P}(a, b) = \max\{\min (n : L^n(G) \text{ has property } \mathcal{P} : \kappa'(G) \geq a \text{ and } \delta(G) \geq b)\}$, and proposed a few problems to determine $\mathcal{P}(a, b)$ when $\mathcal{P}$ is being Hamiltonian, edge-Hamiltonian and Hamiltonian-connected. S. Zhan [ibid. 22, 89–95 (1986; Zbl 0611.05038)] proved that the line graph of a 4-edge-connected graph is Hamilton-connected, which implies a solution to the unsettled cases of above-mentioned problem. We consider an extended version of the problem. Let $\text{ess}'(G)$ denote the essential edge-connectivity of a graph $G$, and define $\mathcal{P}'(a, b) = \max\{\min (n : L^n(G) \text{ has property } \mathcal{P} : \text{ess}'(G) \geq a \text{ and } \delta(G) \geq b)\}$. We investigate the values of $\mathcal{P}'(a, b)$ when $\mathcal{P}$ is one of these Hamiltonian properties. In particular, we show that for any values of $b \geq 1$, $\mathcal{P}'(4, b) \leq 2$ and $\mathcal{P}'(4, b) = 1$ if and only if Thomassen’s conjecture that every 4-connected line graph is Hamiltonian is valid [C. Thomassen, J. Graph Theory 10, No. 3, 309–324 (1986; Zbl 0614.05050)].

MSC:

05C45 Eulerian and Hamiltonian graphs
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
05C40 Connectivity
05C76 Graph operations (line graphs, products, etc.)

Keywords:

edge-connectivity; spanning tree packing; Hamiltonian line graph; Hamilton-connected line graph; Hamiltonian index

Full Text: DOI

References:

[14] Kučzel, R.; Xiong, L., Every 4-connected Line Graph is Hamiltonian if and only if it is Hamiltonian Connected, in R. Kučzel: Hamiltonian properties of graphs (2004), U.W.B. Pilsen, Ph.D. Thesis

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Zhan, S. M., Hamiltonian-connectedness of line graphs, Ars Comb., 22, 89-95 (1986) - Zbl 0611.05038
