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Left dihedral codes over finite chain rings. (English) Zbl 07473530
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Summary: Let $R$ be a finite commutative chain ring, $D_{2n}$ be the dihedral group of size $2n$ and $R[D_{2n}]$ be the dihedral group ring. In this paper, we completely characterize left ideals of $R[D_{2n}]$ (called left $D_{2n}$-codes) when $\gcd(\text{char}(R), n) = 1$. In this way, we explore the structure of some skew-cyclic codes of length 2 over $R$ and also over $R \times S$, where $S$ is an isomorphic copy of $R$. As a particular result, we give the structure of cyclic codes of length 2 over $R$. In the case where $R = \mathbb{F}_{p^m}$ is a Galois field, we give a classification for left $D_{2N}$-codes over $\mathbb{F}_{p^m}$, for any positive integer $N$. In both cases we determine dual codes and identify self-dual ones.

MSC:
94Bxx Theory of error-correcting codes and error-detecting codes
11Txx Finite fields and commutative rings (number-theoretic aspects)
20Cxx Representation theory of groups

Keywords:
left dihedral codes; chain rings; skew-cyclic codes; automorphism; dual codes; self-dual codes

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