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Hamiltonian s-properties and eigenvalues of k-connected graphs. (English) [Zbl 07473531]
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Summary: V. Chvátal and P. Erdős [ibid. 2, 111–113 (1972; Zbl 0233.05123)] proved that, for a k-connected graph \( G \), if the stability number \( \alpha(G) \leq k - s \), then \( G \) is Hamilton-connected \((s = 1)\) or Hamiltonian \((s = 0)\) or traceable \((s = -1)\). Motivated by the result, we focus on tight sufficient spectral conditions for k-connected graphs to possess Hamiltonian s-properties. We say that a graph possesses Hamiltonian s-properties, which means that the graph is Hamilton-connected if \( s = 1 \), Hamiltonian if \( s = 0 \), and traceable if \( s = -1 \).

For a real number \( a \geq 0 \), and for a \( k \)-connected graph \( G \) with order \( n \), degree diagonal matrix \( D(G) \) and adjacency matrix \( A(G) \), we have identified best possible upper bounds for the spectral radius \( \lambda_1(aD(\Gamma) + A(\Gamma)) \), where \( \Gamma \) is either \( G \) or the complement of \( G \), to warrant that \( G \) possesses Hamiltonian s-properties.

Sufficient conditions for a graph \( G \) to possess Hamiltonian s-properties in terms of upper bounds for the Laplacian spectral radius as well as lower bounds of the algebraic connectivity of \( G \) are also obtained.

Other best possible spectral conditions for Hamiltonian s-properties are also discussed.

MSC:
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
05C40 Connectivity
05C45 Eulerian and Hamiltonian graphs
15A42 Inequalities involving eigenvalues and eigenvectors

Keywords:
k-connected graphs; Hamiltonian s-properties; eigenvalues; quotient matrix

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References: