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Symbolic powers of monomial ideals. (English) Zbl 07475386
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Summary: Let $K$ be a field and consider the standard grading on $A = K[X_1, \ldots, X_d]$. Let $I, J$ be monomial ideals in $A$. Let $I_n(J) = (I^n : J^\infty)$ be the $n$th symbolic power of $I$ with respect to $J$. It is easy to see that the function $f^J_I(n) = c_0(I_n(J)/I^n)$ is of quasi-polynomial type, say of period $g$ and degree $c$. For $n \gg 0$ say
\[ f^J_I(n) = a_c(n)n^c + a_{c-1}(n)n^{c-1} + \text{lower terms}, \]
where for $i = 0, \ldots, c$, $a_i : \mathbb{N} \to \mathbb{Q}$ are periodic functions of period $g$ and $a_c \neq 0$. In J. Herzog et al. [Math. Proc. Camb. Philos. Soc. 145, No. 3, 623–642 (2008; Zbl 1157.13013)] proved that $\dim I_n(J)/I^n$ is constant for $n \gg 0$ and $a_c(\cdot)$ is a constant. In this paper we prove that if $I$ is generated by some elements of the same degree and height $I \geq 2$ then $a_{c-1}(\cdot)$ is also a constant.

MSC:
13D40 Hilbert-Samuel and Hilbert-Kunz functions; Poincaré series
13H15 Multiplicity theory and related topics

Keywords:
quasi-polynomials; monomial ideals; symbolic powers

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