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Let $K$ be a compact subset of $\mathbb{C}^N$, let $h_s = \binom{s+N-1}{N-1}$, and let $\{e_{s,i}(z)\}$ (where $1 \leq i \leq h_s$) be the set of all monomials $z^{\alpha_1} \cdots z^{\alpha_N}$ of degree $s$ ordered lexicographically. The author proves that

$$\lim_{s \to \infty} \left( \sup \{ \det [e_{s,i}(x_j)]_{1 \leq i,j \leq k} : x_1, \ldots, x_k \in K, 1 \leq k \leq h_s \} \right)^{1/(sh_s)}$$

exists, answering a question of J. Siciak [Trans. Am. Math. Soc. 105, 322-357 (1962; Zbl 0111.081)]. This limit is called the homogeneous transfinite diameter of $K$, denoted by $D(K)$. Finally some properties of $D(K)$ are obtained, and its value calculated for several sets $K$; for example, if

$$K = \{(z_1, \ldots, z_N) \in \mathbb{C}^N : |z_1| \leq R_1, \ldots, |z_N| \leq R_N \}$$

then $D(K) = \left( \prod_{j=1}^N R_j \right)^{1/N}$.

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MSC:

31C15 Potentials and capacities on other spaces
32A30 Other generalizations of function theory of one complex variable
30C85 Capacity and harmonic measure in the complex plane

Cited in 3 Documents

Keywords:
Chebyshev constant; homogeneous polynomials; extremal points; compact subset; homogeneous transfinite diameter

Full Text: DOI