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Asymptotic distribution of the zeros of recursively defined non-orthogonal polynomials.
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Summary: Let \( g \) be a normalized arithmetic function. We define polynomials
\[
Q^n_g(x) = x \sum_{k=1}^{n} g(k)Q^n_{g-k}(x), \quad Q^0_g(x) := 1.
\]

It is known that the case \( g = \text{id} \) involves Chebyshev polynomials of the second kind \( Q^2_{g-n}(x) = xU_{n-1}(\frac{x}{2}+1) \).
In this paper we study the zero distribution of the non-orthogonal polynomials associated with \( s(n) = n^2 \). We show that the zeros of \( Q^n_s(x) \) are real, simple, and are located in \((-6\sqrt{3}, 0)\). Let \( N_n(a, b) \) be the number of zeros between \(-6\sqrt{3} \leq a < b \leq 0\). Then we determine a density function \( v(x) \), such that
\[
\lim_{n \to \infty} \frac{N_n(a, b)}{n} = \int_a^b v(x) \, d x.
\]

The polynomials \( Q^n_s(x) \) satisfy a four-term recursion. We present in detail an analysis of the fundamental roots and give an answer to an open question on recent work by Adams and Tran-Zumba. We extend a method proposed by Freud for orthogonal polynomials to more general systems of polynomials. We determine the underlying moments and density function for the zero distribution.

MSC:
41-XX Approximations and expansions
42-XX Harmonic analysis on Euclidean spaces
11B37 Recurrences
30C15 Zeros of polynomials, rational functions, and other analytic functions of one complex variable (e.g., zeros of functions with bounded Dirichlet integral)
26C10 Real polynomials: location of zeros
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords: moments; polynomials; recurrence; zero distribution

Full Text: DOI

References:

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