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Menger-bounded groups and axioms about filters. (English) Zbl 07481217

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Summary: A topological group G is *Menger-bounded* if, for each sequence $U_1, U_2, \dots$ of open sets, there are finite sets $F_1, F_2, \dots$ such that $G = \bigcup_n F_n \cdot U_n$. It is *Scheepers-bounded* if all of its finite powers are Menger-bounded. A notorious open problem asks whether, consistently, every product of two Menger-bounded subgroups of the Baer-Specker group $\mathbb{Z}^{\mathbb{N}}$ is Menger-bounded. We prove that the same assertion for Scheepers-bounded groups is equivalent to the set-theoretic axiom NCF (Near Coherence of Filters). We also show that Menger-bounded *sets* are not productive, and that the preservation of Scheepers-bounded subsets of $[\mathbb{N}]^\omega$ by finite-to-one quotients is equivalent to nonexistence of rapid filters.

MSC:

03E17 Cardinal characteristics of the continuum
26A03 Foundations: limits and generalizations, elementary topology of the line
03E75 Applications of set theory

Keywords:

Menger-bounded set; Scheepers-bounded set; NCF; rapid filter

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