Menger-bounded groups and axioms about filters.

Summary: A topological group $G$ is Menger-bounded if, for each sequence $U_1, U_2, \ldots$ of open sets, there are finite sets $F_1, F_2, \ldots$ such that $G = \bigcup_n F_n \cdot U_n$. It is Scheepers-bounded if all of its finite powers are Menger-bounded. A notorious open problem asks whether, consistently, every product of two Menger-bounded subgroups of the Baer-Specker group $Z^N$ is Menger-bounded. We prove that the same assertion for Scheepers-bounded groups is equivalent to the set-theoretic axiom NCF (Near Coherence of Filters). We also show that Menger-bounded sets are not productive, and that the preservation of Scheepers-bounded subsets of $[\mathbb{N}]^{<\omega}$ by finite-to-one quotients is equivalent to nonexistence of rapid filters.

MSC:

03E17  Cardinal characteristics of the continuum
26A03  Foundations: limits and generalizations, elementary topology of the line
03E75  Applications of set theory

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References:


