Ricceri, Biagio
An improvement of a saddle point theorem and some of its applications. (English)

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Summary: We establish an improved version of a saddle point theorem ([the author, J. Nonlinear Var. Anal. 4, No. 1, 21–26 (2020; Zbl 1443.49020)]) removing a weak lower semicontinuity assumption at all. We then revisit some of the applications of that theorem in the light of such an improvement. For instance, we obtain the following very general result of local nature: Let \((H, \langle \cdot, \cdot \rangle)\) be a real Hilbert space and \(\Phi : B_r \to H\) a \(C^{1,1}\) function, with \(\Phi(0) \neq 0\). Then, for each \(r > 0\) small enough, there exist only two points \(x^*, u^* \in S_r\), such that

\[
\max \{\langle \Phi(x^*), x^* - x \rangle, \langle \Phi(x), x^* - x \rangle\} < 0,
\]

for all \(x \in B_r \setminus \{x^*\}\),

\[
\|\Phi(u^*) - u^*\| = \text{dist}(\Phi(u^*), B_r)
\]

and

\[
\|\Phi(x) - u^*\| < \|\Phi(x) - x\|
\]

for all \(x \in B_r \setminus \{u^*\}\), where

\[
B_r = \{x \in H : \|x\| \leq r\}
\]

and

\[
S_r = \{x \in H : \|x\| = r\}.
\]

MSC:
41A50 Best approximation, Chebyshev systems
41A52 Uniqueness of best approximation
47J20 Variational and other types of inequalities involving nonlinear operators (general)
49J35 Existence of solutions for minimax problems
49J40 Variational inequalities

Keywords:
saddle point; Hilbert space; ball; \(C^{1,1}\) function; variational inequality; best approximation point

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