Let $G$ be a topological group and let $P \to M$ be a principal $G$-bundle over a manifold $M$. The isomorphism class of $P$ is uniquely determined by the homotopy class of its inducing map $\alpha : M \to BG$. The gauge group of $P$, denoted by $\mathcal{G}_\alpha(M)$, is the topological group consisting of $G$-equivariant automorphisms of $P$ that fix $M$. The study of the topology of gauge groups is closely related to hot topics in mathematical physics and the geometry of manifolds.

In this paper the author considers three types of high dimensional manifolds $M$, and studies the homotopy types of gauge groups of their principal $G$-bundles. The three types of high dimensional manifolds are: $(n-1)$-connected closed oriented combinatorial $2n$-manifolds, the total spaces of oriented sphere bundles of real vector bundles over spheres with cross sections, and highly connected closed oriented $2n$-manifolds. The author shows that under certain conditions a wedge decomposition of $\Sigma M$ induces a product decomposition of gauge groups of principal $G$-bundles over $M$ (Proposition 2.2). Since suspensions of the three types of manifolds are homotopy equivalent to wedges of smaller spaces, their gauge groups $\mathcal{G}_\alpha(M)$ are homotopy equivalent to products of recognizable spaces (Theorems 1.1–1.3).

In Section 7 the author applies the gauge group decomposition to computing the homotopy exponents of $\mathcal{G}_\alpha(M)$. Given a prime $p$ the homotopy exponent $\text{exp}_p(\mathcal{G}_\alpha(G))$ is the least power of $p$ annihilating all $p$-torsions in $\pi_*(\mathcal{G}_\alpha(G))$. Using Theorems 1.1-1.3 the author gives upper bounds to $\text{exp}_p(\mathcal{G}_\alpha(M))$ (Lemma 7.1), and sharpens the estimates when the structural group $G$ is a certain low rank Lie group with respect to $p$ (Propositions 7.6 and 7.7).

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MSC:

55P15 Classification of homotopy type
55P40 Suspensions
54C35 Function spaces in general topology
55R25 Sphere bundles and vector bundles in algebraic topology
57R19 Algebraic topology on manifolds and differential topology
57S05 Topological properties of groups of homeomorphisms or diffeomorphisms

Keywords:

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