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Poisson-commutative subalgebras of $S(\mathfrak{g})$ associated with involutions. (English) Zbl 07487112


Summary: The symmetric algebra $S(\mathfrak{g})$ of a reductive Lie algebra $\mathfrak{g}$ is equipped with the standard Poisson structure, that is, the Lie-Poisson bracket. Poisson-commutative subalgebras of $S(\mathfrak{g})$ attract a great deal of attention because of their relationship to integrable systems and, more recently, to geometric representation theory. The transcendence degree of a Poisson-commutative subalgebra $\mathcal{C} \subset S(\mathfrak{g})$ is bounded by the “magic number” $b(\mathfrak{g})$ of $\mathfrak{g}$. There are two classical constructions of $\mathcal{C}$ with $\text{tr.deg} \mathcal{C} = b(\mathfrak{g})$. The 1st one is applicable to $\mathfrak{gl}_n$ and $\mathfrak{so}_n$ and uses the Gelfand-Tsetlin chains of subalgebras. The 2nd one is known as the “argument shift method” of Mishchenko-Fomenko. We generalise the Gelfand-Tsetlin approach to chains of almost arbitrary symmetric subalgebras. Our method works for all types. Starting from a symmetric decompositions $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$, Poisson-commutative subalgebras $\mathcal{Z}, \mathcal{Z}' \subset S(\mathfrak{g})^{\mathfrak{g}_0}$ of the maximal possible transcendence degree are constructed. If the $\mathbb{Z}_2$-contraction $\mathfrak{g}_0 \ltimes \mathfrak{g}_1^{\mathfrak{g}_0}$ has a polynomial ring of symmetric invariants, then $\mathcal{Z}'$ is a polynomial maximal Poisson-commutative subalgebra of $S(\mathfrak{g})^{\mathfrak{g}_0}$ and its free generators are explicitly described.

MSC:

17B63 Poisson algebras
17B08 Coadjoint orbits; nilpotent varieties
17B22 Root systems
22E46 Semisimple Lie groups and their representations

Full Text: DOI