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Derived categories of singular surfaces. (English) Zbl 07499435


Summary: We develop an approach that allows one to construct semiorthogonal decompositions of derived categories of surfaces with cyclic quotient singularities whose components are equivalent to derived categories of local finite-dimensional algebras.

We first explain how to induce a semiorthogonal decomposition of a surface $X$ with rational singularities from a semiorthogonal decomposition of its resolution. In the case when $X$ has cyclic quotient singularities, we introduce the condition of adherence for the components of the semiorthogonal decomposition of the resolution that allows one to identify the components of the induced decomposition of $X$ with derived categories of local finite-dimensional algebras. Further, we present an obstruction in the Brauer group of $X$ to the existence of such a semiorthogonal decomposition, and show that in the presence of the obstruction a suitable modification of the adherence condition gives a semiorthogonal decomposition of the twisted derived category of $X$.

We illustrate the theory by exhibiting a semiorthogonal decomposition for the untwisted or twisted derived category of any normal projective toric surface depending on whether its Weil divisor class group is torsion-free or not. For weighted projective planes we compute the generators of the components explicitly and relate our results to the results of Kawamata based on iterated extensions of reflexive sheaves of rank 1.

MSC:

14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14J17 Singularities of surfaces or higher-dimensional varieties
14F22 Brauer groups of schemes

Keywords:
derived categories; semiorthogonal decompositions; toric surfaces; Brauer group

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References:


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