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Scattering diagrams from asymptotic analysis on Maurer-Cartan equations. (English)

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Summary: Let \check{X}_0 be a semi-flat Calabi-Yau manifold equipped with a Lagrangian torus fibration $\check{p} : \check{X}_0 \rightarrow B_0$. We investigate the asymptotic behavior of Maurer-Cartan solutions of the Kodaira-Spencer deformation theory on \check{X}_0 by expanding them into Fourier series along fibres of \check{p} over a contractible open subset $U \subset B_0$, following a program set forth by Fukaya [Graphs and Patterns in Mathematics and Theoretical Physics (2005)] in 2005. We prove that semi-classical limits (i.e. leading order terms in asymptotic expansions) of the Fourier modes of a specific class of Maurer-Cartan solutions naturally give rise to consistent scattering diagrams, which are tropical combinatorial objects that have played a crucial role in works of Kontsevich and Soibelman [The Unity of Mathematics (2006)] and Gross and Siebert [Ann. of Math. (2) 174 (2011)] on the reconstruction problem in mirror symmetry.

MSC:

14J33 Mirror symmetry (algebraic-geometric aspects)

32G05 Deformations of complex structures

14T90 Applications of tropical geometry

53D37 Symplectic aspects of mirror symmetry, homological mirror symmetry, and Fukaya category

Keywords:

scattering diagram; Maurer-Cartan equation; deformation theory; mirror symmetry

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