Fractional meanings of nonrepetitiveness. (English) Zbl 07501773

Summary: A sequence $S$ is called $r$-nonrepetitive if no $r$ sequentially adjacent blocks in $S$ are identical. By the classic results of Thue from the beginning of the 20th century, we know that there exist arbitrarily long binary 3-nonrepetitive sequences and ternary 2-nonrepetitive sequences. This discovery stimulated over the years intensive research leading to various generalizations and many exciting problems and results in combinatorics on words.

In this paper, we study two fractional versions of nonrepetitive sequences. In the first one, we demand that all subsequences of a sequence $S$, with gaps bounded by a fixed integer $j \geq 1$, are $r$-nonrepetitive. (This variant emerged from studying nonrepetitive colorings of the Euclidean plane.) Let $\pi_j(r)$ denote the least size of an alphabet guaranteeing existence of arbitrarily long such sequences. We prove that $\left\lceil \frac{j}{r-1} \right\rceil + 1 \leq \pi_j(r) \leq 2 \left\lceil \frac{j}{r-1} \right\rceil + 1$, for all $r \geq 3$ and $j \geq 1$. We also consider a more general situation with the gap bound $j$ being a real number, and apply this to nonrepetitive coloring of the plane. The second variant allows for using a “fractional” alphabet, analogously as for the fractional coloring of graphs. More specifically, we look for sequences of $b$-element subsets $B_1, B_2, \ldots$ of an $a$-element alphabet, with the ratio $a/b$ as small as possible, such that every member of the Cartesian product $B_1 \times B_2 \times \cdots$ is $r$-nonrepetitive. By using the entropy compression argument, we prove that the corresponding parameter $\pi_f(r) = \inf \frac{a}{b}$ can be arbitrarily close to 1 for sufficiently large $r$.

MSC:
68R15 Combinatorics on words
05A05 Permutations, words, matrices
05A15 Exact enumeration problems, generating functions

Keywords:
nonrepetitive sequence; pattern avoidance; fractional coloring

Full Text: DOI

References:
[1] Aberkane, A.; Currie, J. D., There exist binary circular $\left\lceil(5/2^+1)\right\rceil$ power free words of every length, Electron. J. Comb., 11 (2004) · Zbl 1058.68084
This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.