**Kozlov, K. L.**

**Uniform equicontinuity and groups of homeomorphisms.** (English) Zbl 07506884

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Summary: If $G$ is a group of homeomorphisms of a uniform space $(X, \mathcal{L})$ and the action is uniformly equicontinuous, then the topologies of pointwise $\tau_p$ and uniform $\tau_{\mathcal{L}}$ convergences are among admissible group topologies. We investigate uniform properties of topological groups $(G, \tau_p)$ and $(G, \tau_{\mathcal{L}})$ of homeomorphisms of a uniform space $X$ with uniformly equicontinuous action, uniform properties of $X$ and connections between them. If $X$ is a coset space of $G$ with respect to a neutral subgroup and the maximal equiuniformity $\mathcal{U}$ on $X$ is totally bounded, then the action is uniformly micro-transitive. Necessary and sufficient conditions when the group of homeomorphisms in the topology of pointwise convergence is $\kappa$-narrow (in particular precompact) are given. Spectral representations of acting groups and phase spaces are presented. A sufficient condition for the Roelcke precompactness of a topological group is established. For the actions of the unitary group on the unit sphere in a Hilbert space and of the isometry group on the Urysohn sphere $\mathbb{U}_1$ in the topology of pointwise convergence the maximal equiuniformities are totally bounded. The maximal equivariant compactification $\beta_G\mathbb{U}_1$ is homeomorphic to the Hilbert cube.

**MSC:**

54H15 Transformation groups and semigroups (topological aspects)
22F05 General theory of group and pseudogroup actions
54E15 Uniform structures and generalizations
54B15 Quotient spaces, decompositions in general topology
57S05 Topological properties of groups of homeomorphisms or diffeomorphisms



**Keywords:**

topological group; group of homeomorphisms; $G$-space; uniformity; uniformities on a group

**Full Text:** DOI

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