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Truly optimal Euclidean spanners.  (English) [Zbl 07510280]

Summary: Euclidean spanners are important geometric structures, having found numerous applications over the years. Cornerstone results in this area from the late 1980s and early 1990s state that for any \(d\)-dimensional \(n\)-point Euclidean space, there exists a \((1 + \epsilon)\)-spanner with \(n \cdot O(\epsilon^{-d+1})\) edges and lightness (normalized weight) \(O(\epsilon^{-2d})\). Surprisingly, the fundamental question of whether or not these dependencies on \(\epsilon\) and \(d\) for small \(d\) can be improved has remained elusive, even for \(d = 2\). This question naturally arises in any application of Euclidean spanners where precision is a necessity (thus \(\epsilon\) is tiny). In the most extreme case \(\epsilon\) is inverse polynomial in \(n\), and then one could potentially improve the size and lightness bounds by factors that are polynomial in \(n\). The state-of-the-art bounds \(n \cdot O(\epsilon^{-d+1})\) and \(O(\epsilon^{-2d})\) on the size and lightness of spanners are realized by the greedy spanner. In 2016, in a preliminary version, Filtsner and Solomon [SIAM J. Comput., 49 (2020), pp. 429-447] proved that, in low-dimensional spaces, the greedy spanner is “near-optimal”; informally, their result states that the greedy spanner for dimension \(d\) is just as sparse and light as any other spanner but for dimension larger by a constant factor. Hence the question of whether the greedy spanner is truly optimal remained open to date. The contribution of this paper is twofold: (1) We resolve these longstanding questions by nailing down the dependencies on \(\epsilon\) and \(d\) and showing that the greedy spanner is truly optimal. Specifically, for any \(d = O(1), \epsilon = \Omega(n^{-\frac{1}{d+1}})\), (a) we show that there are \(n\)-point sets in \(\mathbb{R}^d\) for which any \((1 + \epsilon)\)-spanner must have \(n \cdot \Omega(\epsilon^{-d+1})\) edges, implying that the greedy (and other) spanners achieve the optimal size; (b) we show that there are \(n\)-point sets in \(\mathbb{R}^d\) for which any \((1 + \epsilon)\)-spanner must have lightness \(\Omega(\epsilon^{-d})\), and then improve the upper bound on the lightness of the greedy spanner from \(O(\epsilon^{-2d})\) to \(O(\epsilon^{-d} \log(\epsilon^{-1}))\). (The lightness upper and lower bounds match up to a lower-order term.) (2) We then complement our negative result for the size of spanners with a rather counterintuitive positive result: Steiner points lead to a quadratic improvement in the size of spanners! Our bound for the size of Steiner spanners in \(\mathbb{R}^2\) is tight as well (up to a lower-order term).

MSC:
68U05  Computer graphics; computational geometry (digital and algorithmic aspects)
68W05  Nonnumerical algorithms
68W40  Analysis of algorithms
68Q25  Analysis of algorithms and problem complexity

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Euclidean spanners; light spanners; Steiner spanners; spherical codes

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