Fucci, Guglielmo; **Gesztesy, Fritz**; **Kirsten, Klaus**; **Littlejohn, Lance L.**; **Nichols, Roger**; **Stanfill, Jonathan**

The Krein-von Neumann extension revisited. (English) Zbl 07513902

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Summary: We revisit the Krein-von Neumann extension in the case where the underlying symmetric operator is strictly positive and apply this to derive the explicit form of the Krein-von Neumann extension for singular, general (i.e., three-coefficient) Sturm-Liouville operators on arbitrary intervals. In particular, the boundary conditions for the Krein-von Neumann extension of the strictly positive minimal Sturm-Liouville operator are explicitly expressed in terms of generalized boundary values adapted to the (possible) singularity structure of the coefficients near an interval endpoint.

MSC:

34B09 Boundary eigenvalue problems for ordinary differential equations
34B24 Sturm-Liouville theory
34C10 Oscillation theory, zeros, disconjugacy and comparison theory for ordinary differential equations
34L40 Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)
34B20 Weyl theory and its generalizations for ordinary differential equations
34B30 Special ordinary differential equations (Mathieu, Hill, Bessel, etc.)

Keywords:

Krein-von Neumann extension; singular Sturm-Liouville operators; Bessel and Jacobi-type differential operators

Software:

DLMF

Full Text: DOI

References:

[1] Faris, WG., Self-adjoint operators, 433 (1975), Berlin: Springer, Berlin

[2] Kato, T., Perturbation theory for linear operators (1980), Berlin: Springer, Berlin

[3] Krein, MG., The theory of self-adjoint extensions of semi-bounded Hermitian transformations and its applications. I, Mat Sbornik, 20, 431-495 (1947) · Zbl 0029.14103

[4] Alonso, A.; Simon, B., The Birman-Krein-Vishik theory of selfadjoint extensions of semibounded operators, J Operator Theory, 4, 251-270 (1980) · Zbl 0467.47017

[5] Arlinski, Yu. M.; Tsekanovski, ER., The von Neumann problem for nonnegative symmetric operators, Integr Equ Oper Theory, 51, 319-356 (2005)

[6] von Neumann, J., Allgemeine Eigenwerttheorie Hermitescher Funktionaloperatoren, Math Ann, 102, 49-131 (192930) · Zbl 55.0824.02

[7] Krein, MG., The theory of self-adjoint extensions of semi-bounded Hermitian transformations and its applications. II, Mat Sbornik, 21, 365-404 (1947)

[8] Gesztesy, F.; Littlejohn, LL; Nichols, R., On self-adjoint boundary conditions for singular Sturm-Liouville operators bounded from below, J Differ Equ, 269, 6448-6491 (2020) · Zbl 1458.34143

[9] Gesztesy, F, Zinchenko, M.Sturm-Liouville Operators, Their Spectral Theory, and Some Applications. in preparation.

[10] Leighton, W.; Morse, M., Singular quadratic functionals, Trans Am Math Soc, 40, 252-286 (1936) · Zbl 62.0577.02

[11] Rellich, F., Die zulässigen Randbedingungen bei den singulären Eigenwertproblemen der mathematischen Physik. (Gewöhnliche Differentialgleichungen zweiter Ordnung.), Math Z, 49, 702-723 (194344) · Zbl 0028.40803

[12] Rellich, F., Halbbeschränkte gewöhnliche Differentialoperatoren zweiter Ordnung, Math Ann, 122, 343-368 (1951) · Zbl 0044.31201

[13] Hartman, P.; Wintner, A., On the assignment of asymptotic values for the solutions of linear differential equations of second

order, Am J Math, 77, 475-483 (1955) · Zbl 0064.33205

[14] Clark, S, Gesztesy, F, Nichols, R. Principal solutions revisited. in Stochastic and Infinite Dimensional Analysis. C. C. Bernido, M. V. Carpio-Bernido, M. Grothaus, T. Kuna, M. J. Oliveira, and J. L. da Silva (eds.), Trends in Mathematics, Birkhäuser, Springer, pp. 85-117. 2016.

[15] Dunford, N.; Schwartz, JT., Linear operators. part II: spectral theory (1988), New York: Wiley Interscience, New York

[16] Hartman, P., Ordinary differential equations (2002), Philadelphia: SIAM, Philadelphia · Zbl 0125.32102

[17] Niessen, H-D; Zettl, A., Singular Sturm-Liouville problems: the Friedrichs extension and comparison of eigenvalues, Proc London Math Soc (3), 64, 545-578 (1992) · Zbl 0768.34015

[18] Zettl, A., Sturm-Liouville theory, 121 (2005), Providence, RI: Amer. Math. Soc., Providence, RI · Zbl 1074.34030

[19] Kalf, H., A characterization of the Friedrichs extension of Sturm-Liouville operators, J London Math Soc (2), 17, 511-521 (1978) · Zbl 0406.34029

[20] Rosenberger, R., A new characterization of the Friedrichs extension of semibounded Sturm-Liouville operators, J London Math Soc (2), 31, 501-510 (1985) · Zbl 0615.34019

[21] Akhiezer, NI; Glazman, IM., Theory of linear operators in Hilbert space, Vol. II (1981), Boston: Pitman, Boston

[22] Coddington, EA; Levinson, N., Theory of ordinary differential equations (1985), Malabar, FL: Krieger Publ., Malabar, FL

[23] Jörgens, K.; Rellich, F., Eigenwerttheorie Gewöhnlicher Differentialgleichungen (1976), Berlin: Springer-Verlag, Berlin

[24] Naimark, MA.Linear Differential Operators. Part II: Linear Differential Operators in Hilbert Space. Transl. by E. R. Dawson, Engl. translation edited by W. N. Everitt, Ungar Publishing, New York. 1968. · Zbl 0227.34020

[25] Pearson, DB., Quantum scattering and spectral theory (1988), London: Academic Press, London

[26] Teschl, G.Mathematical Methods in Quantum Mechanics. With Applications to Schrödinger Operators, 2nd ed., Graduate Studies in Math., Vol. 157, Amer. Math. Soc., RI. 2014. · Zbl 1342.81003

[27] Weidmann, J., Linear operators in Hilbert spaces, 68 (1980), New York: Springer, New York

[28] Weidmann, J., Lineare Operatoren in Hilberträumen. Teil II: Anwendungen (2003), Stuttgart: Teubner, Stuttgart · Zbl 0344.47001

[29] Ando, T.; Nishio, K., Positive selfadjoint extensions of positive symmetric operators, Tohoku Math J (2), 22, 65-75 (1970) · Zbl 0192.47703

[30] Arlinskii, Yu.M., Hassi, S., Sebestyén, Z., de Snoo, H.S.V.: 'On the class of extremal extensions of a nonnegative operator, in Recent Advances in Operator Theory and Related Topics. L. Kérchy, C. Foias, I. Gohberg, and H. Langer (eds.), Operator Theory: Advances and Applications, Vol. 127, Birkhäuser, Basel, 2001, pp. 41-81. · Zbl 0996.47029

[31] Arlinski, Yu., Tsekanovski, E.: 'M.Kren's research on semibounded operators, its contemporary developments, and applications', in Adamyan, V., Berezansky, Y.M., Gohberg, I., Gorbachuk, M.L., Gorbachuk, V., Kochubei, A.N., Langer, H., Popov, G. (Eds.): 'Modern analysis and applications. the Mark Krein centenary conference. Operator theory: advances and applications', vol. 1 (Birkhäuser, Basel, 2009), pp. 65-112.

[32] Ashbaugh, MS; Gesztesy, F.; Mitrea, M.; Teschl, G., Spectral theory for perturbed Krein Laplacians in nonsmooth domains, Adv Math, 223, 1372-1467 (2010) · Zbl 1191.35188

[33] Ashbaugh, MS; Gesztesy, F.; Mitrea, M.; Shterenberg, R.; Teschl, G., The Krein-von Neumann extension and its connection to an abstract buckling problem, Math Nachr, 283, 165-179 (2010) · Zbl 1183.35100

[34] Ashbaugh, MS, Gesztesy, F, Mitrea, M, Shterenberg, R, Teschl, G.A survey on the Krein-von Neumann extension, the corresponding abstract buckling problem, and Weyl-type spectral asymptotics for perturbed Krein Laplacians in non smooth domains. in Mathematical Physics, Spectral Theory and Stochastic Analysis, M. Demuth and W. Kirsch (eds.), Operator Theory: Advances and Applications, Vol. 232, Birkhäuser, Springer, Basel, pp. 1-106. 2013. · Zbl 1283.47001

[35] Ashbaugh, MS; Gesztesy, F.; Laptev, A.; Mitrea, M.; Sukhtaiev, S., A bound for the eigenvalue counting function for Krein-von Neumann and Friedrichs extensions, Adv Math, 304, 1108-1155 (2017) · Zbl 1362.35197

[36] Behrndt, J.; Hassi, S.; de Snoo, H., Boundary value problems, Weyl functions, and differential operators, 105 (2020), Springer: Birkhäuser, Springer · Zbl 1457.47001

[37] Sh. Birman, M., On the theory of self-adjoint extensions of positive definite operators, Mat Sbornik, 38, 431-450 (1956)

[38] Derkach, VA; Malamud, MM., Generalized resolvents and the boundary value problems for Hermitian operators with gaps, J Funct Anal, 95, 1-95 (1991) · Zbl 0748.47004

[39] Derkach, VA; Malamud, MM., The extension theory of Hermitian operators and the moment problem, J Math Sci, 73, 141-242 (1995) · Zbl 0848.47004

[40] Grubb, G., Spectral asymptotics for the "soft" selfadjoint extension of a symmetric elliptic differential operator, J Oper Theory, 10, 9-20 (1983) · Zbl 0559.47035

[41] Grubb, G., Distributions and operators, 252 (2009), New York: Springer, New York · Zbl 1171.47001

[42] Hassi, S.; Malamud, M.; de Snoo, H., On Kren's extension theory of nonnegative operators, Math Nachr, 274-275, 40-73 (2004) · Zbl 1076.47008

[43] Hassi, S.; Sandovici, A.; de Snoo, H.; Winkler, H., A general factorization approach to the extension theory of nonnegative operators and relations, J Operator Theory, 58, 351-386 (2007) · Zbl 1164.47003

[44] Nenciu, G., Applications of the Kren resolvent formula to the theory of self-adjoint extensions of positive symmetric operators, J Operator Theory, 10, 209-218 (1983) · Zbl 0561.47005

[45] Prokaj, V.; Sebestyén, Z., On extremal positive operator extensions, Acta Sci Math (Szeged), 62, 485-491 (1996) · Zbl 0881.47002

[46] Sebestyén, Z.; Sikolya, E., On Krein-von Neumann and Friedrichs extensions, Acta Sci Math (Szeged), 69, 323-336 (2003) · Zbl 1050.47011

[47] Simon, B., The classical moment problem as a self-adjoint finite difference operator, Adv Math, 137, 82-203 (1998) · Zbl 0910.44004

[48] Skau, CF., Positive self-adjoint extensions of operators affiliated with a von Neumann algebra, Math Scand, 44, 171-195 (1979) · Zbl 0414.46041

[49] Storozh, OG., On the hard and soft extensions of a nonnegative operator, J Math Sci, 79, 1378-1380 (1996)

[50] traus, AV., On extensions of a semibounded operator, Sov Math Dokl, 14, 1075-1079 (1973)

[51] Tsekanovskii, ER., Friedrichs and Krein extensions of positive operators and holomorphic contraction semigroups, Funct Anal Appl, 15, 308-309 (1981)

[52] Viik, ML., On general boundary problems for elliptic differential equations, Trudy Moskov Mat Obsc, 1, 187-246 (1952)

[53] Gesztesy, F, Kalton, N, Makarov, K, Tsekanovskii, E.Some applications of operator-valued Herglotz functions. in Operator Theory, System Theory and Related Topics. The Moshe Livsic Anniversary Volume, D. Alpay and V. Vinnikov (eds.), Operator Theory: Advances and Applications, Vol. 123, Birkhäuser, Basel, pp. 271-321. 2001. · Zbl 0991.30020

[54] Clark, S.; Gesztesy, F.; Nichols, R.; Zinchenko, M., Boundary data maps and Krein's resolvent formula for Sturm-Liouville operators on a finite interval, Oper Matrices, 8, 1-71 (2014) · Zbl 1311.34037

[55] Gesztesy, F, Nichols, R, Stanfill, J.A Survey of Some Norm Inequalities. Comp Anal Oper Theo. 2021;15(23). · Zbl 07352441

[56] Kamke, E., Differentialgleichungen. Lösungsmethoden und Lösungen. gewöhnliche Differentialgleichungen (1961), Leipzig: Akademische Verlagsgesellschaft, Leipzig · Zbl 0096.28204

[57] Abramowitz, M.; Stegun, IA., Handbook of mathematical functions (1972), New York: Dover, New York

[58] Belinskiy, BP, Hinton, DB, Nichols, R. Singular Sturm-Liouville operators with extreme properties that generate black holes. Stud Appl Math. 2021:1-29. DOI:. · Zbl 1487.34083

[59] Gesztesy, F, Littlejohn, LL, Piorkowski, M, Stanfill, J.The Jacobi operator and its Weyl-Titchmarsh-Kodaira m-functions. preprint. 2020.

[60] Bush, M, Frymark, D, Liaw, C.Singular boundary conditions for Sturm-Liouville operators via perturbation theory. preprint. 2020.

[61] Everitt, WN.A catalogue of Sturm-Liouville differential equations. in Sturm-Liouville Theory: Past and Present. W. O. Amrein, A. M. Hinz, D. B. Pearson (eds.), Birkhäuser, Basel, pp. 271-331. 2005. · Zbl 1088.34017

[62] Everitt, WN; Kwon, KH; Littlejohn, LL; Wellman, R.; Yoon, GJ., Jacobi Stirling numbers, Jacobi polynomials, and the left-definite analysis of the classical Jacobi differential expression, J Comput Appl Math, 208, 29-56 (2007) · Zbl 1119.33009

[63] Frymark, D., Boundary triples and Weyl m-functions for powers of the Jacobi differential operator, J Differ Equ, 269, 7931-7974 (2020) · Zbl 07216743

[64] Grünewald, U., Jacobische Differentialoperatoren, Math Nachr, 63, 239-253 (1974) · Zbl 0325.34026

[65] Koornwinder, T.; Kostenko, A.; Teschl, G., Jacobi polynomials, Bernstein-type inequalities and dispersion estimates for the discrete Laguerre operator, Adv Math, 333, 796-821 (2018) · Zbl 1405.33012

[66] Kuijlaars, A.; Martinez-Finkelshtein, A.; Orive, R., Orthogonality of Jacobi polynomials with general parameters, Electron Trans Numer Anal, 19, 1-17 (2005) · Zbl 1075.33005

[67] Olver, FW, Lozier, DW, Boisvert, RF, Clark, CW.NIST Handbook of Mathematical Functions, http://dlmf.nist.gov/, Release 1.0.26 of 2020-03-15. · Zbl 1198.00002

[68] Szeg, G.Orthogonal Polynomials. 4th edn, Colloquium Publications, Vol. 23, Amer. Math. Soc., Providence, RI. 1975.

[69] Olver, FWJ, Lozier, DW, Boisvert, RF, Clark, CW (eds.) NIST Handbook of Mathematical Functions. National Institute of Standards and Technology (NIST), U.S. Dept. of Commerce, and Cambridge Univ. Press. 2010.