

**Krupka, Demeter; Janyška, Josef**

**Lectures on differential invariants.** (English) Zbl 0752.53004

Folia Facultatis Scientiarum Naturalium Universitatis Purkynianae Brunensis. Mathematica. 1. Brno: University J. E. Purkyně. 193 p. (1990).

The book consists of two parts. The first one, written by D. Krupka, is devoted to the foundations of the theory of differential invariants. In the second part, J. Janyška uses the general theory for solving concrete problems on finding all natural operators of several types. In the authors' terminology, a differential invariant is an equivariant map between two  $L_n^r$ -spaces, where  $L_n^r$  is the group of all invertible  $r$ -jets from  $\mathbb{R}^n$  into itself with source and target 0. Such maps are in bijection with the natural transformations between two  $r$ -th order natural bundles over  $n$ -manifolds in the sense of A. Nijenhuis. Since the  $k$ -th order natural operators between two natural bundles correspond to the natural transformations from the  $k$ -th jet prolongation of the first bundle into the second one, even the finite order natural operators are in bijection with the related differential invariants.

After some preliminaries on Lie groups and their actions, the jet groups are studied in detail. The first author deduces the coordinate formula for the bracket in the Lie algebra of  $L_n^r$  and applies it for finding all ideals in this Lie algebra,  $r \geq 2$ . Special attention is paid to the jet prolongations of arbitrary principal and associated bundles introduced by C. Ehresmann and to the jet prolongations of natural bundles. Also the standard characterization of equivariant maps in terms of the fundamental vector fields is presented. Then the classical description of all absolute invariant tensors and some properties of the characters of the general linear group and of the relative invariant tensors are explained. They are useful in several concrete problems about naturality.

The second part starts with the infinitesimal characterization of all globally defined homogeneous functions with positive degrees of homogeneity. This is used for finding all natural operators of certain types between the tensor bundles. Then a precise proof of the classical reduction theorem for finite order natural operators on linear torsion-free connections with values in a first order natural bundle is given. Next it is deduced, that the Levi-Civita connection is the only first order natural operator transforming Riemannian metrics into linear connections and some partial results on the higher order operators are added. After that all polynomial lifts of Riemannian metrics to the tangent bundle are determined.

Finally the second author describes all first order natural operators transforming vector fields from a manifold to the tangent bundle, all natural transformations of linear connections into themselves and all first order natural operators, transforming linear connections into principal connections on the second order frame bundle.

Reviewer: [I.Kolář \(Brno\)](#)

**MSC:**

- [53-02](#) Research exposition (monographs, survey articles) pertaining to differential geometry
- [53A55](#) Differential invariants (local theory), geometric objects
- [58A20](#) Jets in global analysis

Cited in **1** Review  
Cited in **23** Documents

**Keywords:**

[finite order natural operators](#); [jet prolongations](#); [polynomial lifts of Riemannian metrics](#)