Asok, Aravind; Fasel, Jean
Euler class groups and motivic stable cohomotopy (with an appendix by Mrinal Kanti Das).
(English) Zbl 07523090 J. Eur. Math. Soc. (JEMS) 24, No. 8, 2775-2822 (2022)

Summary: We study maps from a smooth scheme to a motivic sphere in the Morel-Voevodsky
$A_1$-homotopy category, i.e., motivic cohomotopy sets. Following Borsuk, we show that, in the presence
of suitable hypotheses on the dimension of the source, motivic cohomotopy sets can be equipped
with functorial abelian group structures. We then explore links between motivic cohomotopy groups,
Euler class groups à la Nori-Bhatwadekar-Sridharan and Chow-Witt groups. We show that, again under suitable
hypotheses on the base field $k$, if $X$ is a smooth affine $k$-variety of dimension $d$, then the Euler class group
of codimension $d$ cycles coincides with the codimension $d$ Chow-Witt group; the identification proceeds
by comparing both groups with a suitable motivic cohomotopy group. As a byproduct, we describe
the Chow group of zero cycles on a smooth affine $k$-scheme as the quotient of the free abelian group on zero
cycles by the subgroup generated by reduced complete intersection ideals; this answers a question of S.
Bhatwadekar and R. Sridharan.

MSC:
14F42 Motivic cohomology; motivic homotopy theory
13C10 Projective and free modules and ideals in commutative rings
19A13 Stability for projective modules
55Q55 Cohomotopy groups

Keywords:

motivic homotopy; Euler class; projective modules

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.