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Summary: We explore a new way to handle flux boundary conditions imposed on level sets. The proposed approach is a diffuse interface version of the shifted boundary method (SBM) for continuous Galerkin discretizations of conservation laws in embedded domains. We impose the interface conditions weakly and approximate surface integrals by volume integrals. The discretized weak form of the governing equation has the structure of an immersed boundary finite element method. That is, integration is performed over a fixed fictitious domain. Source terms are included to account for interface conditions and extend the boundary data into the complement of the embedded domain. The calculation of these extra terms requires (i) construction of an approximate delta function and (ii) extrapolation of embedded boundary data into quadrature points. We accomplish these tasks using a level set function, which is given analytically or evolved numerically. A globally defined averaged gradient of this approximate signed distance function is used to construct a simple map to the closest point on the interface. The normal and tangential derivatives of the numerical solution at that point are calculated using the interface conditions and/or interpolation on uniform stencils. Similarly to SBM, extrapolation of data back to the quadrature points is performed using Taylor expansions. Computations that require extrapolation are restricted to a narrow band around the interface. Numerical results are presented for elliptic, parabolic, and hyperbolic test problems, which are specifically designed to assess the error caused by the numerical treatment of interface conditions on fixed and moving boundaries in 2D.

MSC:
65Nxx Numerical methods for partial differential equations, boundary value problems
76Mxx Basic methods in fluid mechanics
65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems

Keywords: 
unfitted finite element method; level set algorithm; diffuse interface; regularized delta function; ghost penalty; extension velocity

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