Uļ’yanov, P. L.
On properties of functions in the Gevrey classes. (English. Russian original) Zbl 0753.42005

In this note, the author has given seven theorems without proof. These theorems are concerned with certain subspaces of the space $C^\infty$, the space of all $2\pi$-periodic real functions $f$ having infinitely many derivatives with Fourier coefficients $a_n(f)$. One of the subspaces of $C^\infty$ is the Gevrey class of functions.

To have a flavour of these theorems we define:

$$J(\alpha, m)(\alpha > 0, m > 0) = \{ f \in C^\infty \mid \| f^{(k)} \|_\infty \leq A(f)m^k(k!)^\alpha, k \geq 1 \},$$

where $f^{(k)}$ and $\| \cdot \|_\infty$, respectively, denote $k$-th derivative and sup-norm and the constant $A(f)$ depends upon on $f$. Author’s first theorem reads as follows:

(i) If $f \in J(\alpha, m)$, then

$$|a_n(f)| \leq A(f)C_1(\alpha, m)|n|^{1/2}\theta_1^{1/n}$$

for $|n| \geq 1$,

where $C_1(\alpha, m)$ is a positive constant depending upon $\alpha$ and $m$.

(ii) There exists $f_0 \in C^\infty$ such that $f_0 \in J(\alpha, m)$ but

$$\lim_{|n| \to \infty} \frac{|a_n(f_0)|}{|n|^{1/2}\theta_1^{1/n}} > 0.$$

Reviewer: P. Chandra (Ujjain)

MSC:

42A16 Fourier coefficients, Fourier series of functions with special properties, special Fourier series
41A50 Best approximation, Chebyshev systems
26E10 $C^\infty$-functions, quasi-analytic functions

Keywords:

$C^\infty$-functions; space $C^\infty$; $2\pi$-periodic real functions; Fourier coefficients; Gevrey class