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On sharpness of an error bound for deep ReLU network approximation. (English)  

Summary: Zuowei Shen, Haizhao Yang, and Shijun Zhang recently proved a sharp estimate in terms of a modulus of continuity for the error of approximating continuous functions with ReLU-activated deep neural networks in their paper “Optimal approximation rate of ReLU networks in terms of width and depth”, Journal de Mathématiques Pures et Appliquées (2021). The sharpness was established based on a Vapnik-Chervonenkis dimension estimate by showing that for each choice of certain fixed width and depth parameters, no general smaller error bound applies. This leads to counterexamples that rule out smaller error bounds. In principle, the obtained counterexamples can be different for different parameters. However, for a given convergence rate (e.g., determined by a Lipschitz class), the counterexamples can be condensed to a single one using a quantitative variant of the uniform boundedness principle. Such theorems were developed at the institute of Paul Butzer at RWTH Aachen. When network width or depth tends to infinity in certain cases, the condensed counterexamples show that the approximation order is not in the little-o class of the modulus of continuity used in the error estimate, i.e., the convergence cannot be faster than stated.

MSC:
41A25 Rate of convergence, degree of approximation  
41A50 Best approximation, Chebyshev systems  
62M45 Neural nets and related approaches to inference from stochastic processes

Keywords:
nearal networks; sharpness of error bounds; counterexamples; rates of convergence; uniform boundedness principle

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References:


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